kd education academy (9582701166

Time : 10 Hour

STD 11 Maths

kd90+ ch- 8 sequences and series

Total Marks : 350

*** Choose the right answer from the given options. [1 Marks Each]** **[64]**

1. If 3rd term of an A.P. is 6 and 5th term of that A.P. is 12. Then find the 21st term of that A.P.

(A) 40 (B) 42 (C) 60 (D) 63

Ans. :

c. 60

Solution:

Given, $a_3 = 6$ and $a_5 = 12$.

$\Rightarrow a + 2d = 6$ and $a + 4d = 12$

$\Rightarrow 2d = 6 \Rightarrow d = 3$

2. The sum of first three terms of a G.P. is $\frac{21}{2}$ and their product is 27. Find the common ratio.

(A) 2 (B) $\frac{1}{2}$ (C) $2 \text{ or} \frac{1}{2}$ (D) $\text{neither } 2 \text{ nor} \frac{1}{2}$

Ans. :

c. $2 \text{ or} \frac{1}{2}$

Solution:

Let three terms be $\frac{a}{r}$ a, a × r.

Product $= 27 \Rightarrow (\frac{a}{r})$ (a) $(a \times r) = 27 \Rightarrow a^3 = 27$

$\Rightarrow a = 3.$

$\text{sum} = \frac{21}{2} \Rightarrow \frac{(a}{r + a \times r)} = \frac{21}{2} \Rightarrow a\left(\frac{1}{r+1+1 \times r}\right) = \frac{21}{2}$

$\Rightarrow \left(\frac{1}{r+1+1 \times r}\right)$

$\Rightarrow \left(r^2 + r + 1\right) = \left(\frac{7}{2}\right) \Rightarrow r^2 - \left(\frac{5}{2}\right)r + 1 = 0$

$\Rightarrow r = 2 \text{ and } \frac{1}{2}.$

3. If in an infinite G.P., first term is equal to 10 times the sum of all successive terms, the its common ratio is:

(A) $\frac{1}{10}$ (B) $\frac{1}{11}$ (C) $\frac{1}{9}$ (D) $\frac{1}{20}$

Ans. :

b. $\frac{1}{11}$

Solution:

Let the first term of the G.P. be a.

Let its common ratio be r.

According to the question, we have:

First term = 10 [Sum of all successive terms]

$a = 10\left(\frac{ar}{1-r}\right)$

$\Rightarrow a - ar = 10ar$

$\Rightarrow 11ar = a$

$\Rightarrow r = \frac{a}{11a} = \frac{1}{11}$

4. If an A.P. is 1,7,13, 19, ... Find the sum of 22 terms.

(A) 127 (B) 1204 (C) 1408 (D) 1604

Ans. :

c. 1408

Solution:

From the given A.P., a = 1 and d = 7 - 1 = 6.

We know, $s_n = \frac{n}{2}(2a + (n-1)d$ $s_{22} = \frac{22}{2}(2 \times 1 + (22-1)6)$

= 11(2 + 126) = 11 × 128 = 1408.

5. After striking the floor, a certain ball rebounds $(\frac{4}{5})$ th of height from which it has fallen. Then, the total distance that it travels before coming to rest, if it is gentlydropped from a height of 120 m is:

(A) 1260 m (B) 600 m (C) 1080 m (D) None of these

Ans. :

c. 1080 m

6. If a, b, c are in G.P. is 2 and x, y are AM's between a, b and b, c respectively, then:

(A) $\frac{1}{x} + \frac{1}{y} = 2$ (B) $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$ (C) $\frac{1}{x} + \frac{1}{y} = \frac{2}{a}$ (D) $\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$.

Ans. :

d. $\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$.

Solution:

a, b and c are in G.P.

$\therefore b^2 = ac \cdots (i)$

a, x and b are in A.P.

$\therefore 2x = a + b \cdots (ii)$

Also, b, y and c are in A.P.

$\therefore 2y = b + c$

$\Rightarrow 2y = b + \frac{b^2}{a}$ [Using (i)]

$\Rightarrow 2y = b + \frac{b^2}{(2x-b)}$ [Using (ii)]

$\Rightarrow 2y = b + \frac{b(2x-b)+b^2}{(2x-b)}$

$\Rightarrow 2y = \frac{2bx-b^2+b^2}{(2x-b)}$

$\Rightarrow 2y = \frac{2bx}{(2x-b)}$

$\Rightarrow y = \frac{bx}{(2x-b)}$

$\Rightarrow y(2x-b) = bx$

$\Rightarrow 2xy - by = bx$

$\Rightarrow bx + by = 2xy$

Dividing both the sides by xy:

$\Rightarrow \frac{1}{y} + \frac{1}{x} = \frac{2}{b}$

7. Let so $= \frac{8}{5} + \frac{16}{65} \ldots\ldots + \frac{128}{2}^{18} + 1$:

(A) $s = \frac{1088}{545}$ (B) $s = \frac{1088}{545}$ (C) $s = \frac{1056}{545}$ (D) $s = \frac{545}{1056}$

Ans. :

a. $s = \frac{1088}{545}$

8. The sum of the series $\frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + \ldots + \frac{1}{\log_2 {}^{n}4}$ is:

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(n+1)(2n+1)}{12}$ (C) $\frac{n(n+1)}{4}$ (D) none of these.

Ans. :

c. $\frac{n(n+1)}{4}$

Solution:

Let $S_n = \frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + \ldots + \frac{1}{\log_2 {}^{n}4}$

$\Rightarrow S_n = \frac{\log 2}{\log 4} + \frac{\log 4}{\log 4} + \frac{\log 8}{\log 4} + \ldots + \frac{\log 2^n}{\log 4}$

$\Rightarrow S_n = \frac{\log 2}{\log 4} + \frac{\log 2^2}{\log 4} + \frac{\log 2^3}{\log 4} + \ldots + \frac{\log 2^n}{\log 4}$

$\Rightarrow S_n = \frac{\log 2}{\log 4} + \frac{2\log 2}{\log 4} + \frac{3\log 2}{\log 4} + \ldots + \frac{n\log 2}{\log 4}$

$\Rightarrow S_n = \frac{\log 2}{\log 4}(1 + 2 + 3 + \ldots + n)$

$\Rightarrow S_n = \frac{\log 4^{\frac{1}{2}}}{\log 4}(1 + 2 + 3 + \ldots + n)$

$\Rightarrow S_n = \frac{\frac{1}{2}\log 4}{\log 4}(1 + 2 + 3 + \ldots + n)$

$\Rightarrow S_n = \frac{1}{2}(1 + 2 + 3 + \ldots + n)$

$\Rightarrow S_n = \frac{n(n+1)}{4}$

9. Find the sum of squares of first n terms.

(A) $\frac{n(n+1)}{2}$ (B) $\left(\frac{n(n+1)}{2}\right)^3$ (C) $\frac{n(n+1)(2n+1)}{6}$ (D) $\left(\frac{n(n+1)}{2}\right)$

Ans. :

c. $\frac{n(n+1)(2n+1)}{6}$

Solution:

Sum of squares of first n terms = $1^2+2^2+3^2+$..............$+n^2$

$k^3-(k-1)^3=3k^2-3k+1$

On substituting k = 1, 2, 3, ..., n and adding we get,

$n^3 = \sum_{i}^{n} = 0\ k^2 = \sum_{i}^{n} = 0\ k + n$

$n^3 = 3\sum_{i}^{2} = 0\ k^2 - 3\frac{n(n+1)}{2} + n$

$\sum_{i}^{n} = 0\ k^2 = \frac{n(n+1)(n+2)}{6}.$

10. The nth term of a G.P. is 128 and the sum of its n terms is 225. If its common ratio is 2, then its first term is:

(A) 1 (B) 3 (C) 8 (D) None of these.

Ans. :

a. 1

Solution:

Let the firt term of the geometric progression = x

Common ration = 2

∴ 2nd term of the G.P. = 2x

∴ 3rd term = (2^2)x ...

N th term can be written as $= (2^n)x$

Sum of the n terms S = 255

as we can see, except x, all other terms in the G.P. are multiples of 2 and sum of all the terms is an odd number.

∴ x must be an odd number.

now n^{th} term

$(2^n)x = 128 = (2^7) \times 1$

There are no factors of odd numbers in 128, except 1

∴ x = 1

Series of G.P. is:

1, 2, 4, 8, 16, 32, 64, 128

Checking the sum of the n terms,

$1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 = 255$

$\therefore$ First term of the G.P. = 1

11. If first term of a G.P. is 20 and common ratio is 4. Find the 5^{th} term.

(A) 10240 (B) 40960 (C) 5120 (D) 2560

Ans. :

c. 5120

solution:

Given, a = 20 and r = 4.

We know, $a_n = ar^{n-1}$

$\Rightarrow a_5 = 20 \times 4^4 = 20 \times 256 = 5120.$

12. If A.M. of two numbers is $\frac{15}{2}$ and their G.M. is 6, then find the two numbers.

(A) 6 and 8 (B) 12 and 3 (C) 24 and 6 (D) 27 and 3

Ans. :

b. 12 and 3

Solution:

We know, A.M. of two numbers a and b is

$\frac{(a+b)}{2}$

$\Rightarrow \frac{(a+b)}{2} = \frac{15}{2} \Rightarrow a + b = 15.$

Also, G.M. of two numbers a and b is $\sqrt{ab}$

$\Rightarrow \sqrt{ab} = 6 \Rightarrow ab = 36.$

=> a(15-a) = 36 => a=3 or 12.

For a=3, b=12.

For a=12, b=3.

So, the two numbers are 3 and 12.

13. If $S_n = \sum_{r=1}^{n} \frac{1+2+2^2+ \ ... \text{ Sum to r terms}}{2^r}$, then S_n is equal to:

(A) $2^n - n - 1$ (B) $1 - \frac{1}{2^n}$ (C) $n-1 - \frac{1}{2^n}$ (D) $2^n - 1$

Ans. :

c. $n-1 - \frac{1}{2^n}$

Solution:

We have,

$$S_n = \sum_{r=1}^{n} \frac{1+2+2^2+ \ ... \text{ Sum to r terms}}{2^r}$$

$$\Rightarrow S_n = \sum_{r=1}^{n} \frac{1(2^r - 1)}{2r}$$

$$\Rightarrow S_n = \sum_{r=1}^{n} \left(1 - \frac{1}{2^r}\right)$$

$$\Rightarrow S_n = n - \sum_{r=1}^{n} \left(\frac{1}{2^r}\right)$$

$$\Rightarrow S_n = n - \left[\frac{\left(\frac{1}{2}\right)\left\{1 - \left(\frac{1}{2}\right)^n\right\}}{1 - \frac{1}{2}}\right]$$

$$\Rightarrow S_n = n - \left[1 - \left(\frac{1}{2}\right)^n\right]$$

$$\Rightarrow S_n = n - 1 + \frac{1}{2^n}$$

14. Find the sum of series $1^2 + 3^2 + 5^2 + \ldots\ldots\ldots\ldots + 11^2$.

(A) 279 (B) 286 (C) 309 (D) 409

Ans. :

b. 286

solution:

$1^2 + 3^2 + 5^2 + \ldots\ldots\ldots\ldots + 11^2$

$= (1^2 + 2^2 + 3^2 + \ldots\ldots + 11^2) - (2^2 + 4^2 + 6^2 + 8^2 + 10^2)$

$= (1^2 + 2^2 + 3^2 + \ldots\ldots 11^2) - 2^2(1^2 + 2^2 + 3^2 + 4^2 + 5^2)$

$$= \frac{16 \times 12 \times 23}{6} - \frac{4 \times 5 \times 6 \times 11}{6}$$

$$= 506 - 220 = 286.$$

15. Find the sum of series $6^2 + 7^2 + \ldots\ldots\ldots\ldots + 15^2$.

(A) 55 (B) 1185 (C) 1240 (D) 1385

Ans. :

b. 1185

Solution:

$6^2 + 7^2 + \ldots\ldots\ldots\ldots + 15^2$

$= (1^2 + 2^2 + 3^2 + \ldots\ldots + 15^2) - (1^2 + 2^2 + 3^2 + 4^2 + 5^2)$

$$= \frac{15 \times 16 \times 31}{6} - \frac{5 \times 6 \times 11}{6}$$

$$= 1240 - 55 = 1185.$$

16. The sum of an infinite G.P. is 4 and the sum of the cubes of its terms is 92. The common ratio of original G.P. is:

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{-1}{2}$.

Ans. :

a. $\frac{1}{2}$

Solution:

$\frac{a}{1-r} = 3$

$a = 3 - 3r$

Sum of square terms of G.P. is $\frac{a^2}{1-r^2} = 3$

$\Rightarrow \frac{a}{1-r} = \frac{a^2}{1-r^2}$

or $a = 1 + r \ldots(2)$

Solving (1) and (2),

$a = \frac{3}{2}$ and $r = \frac{1}{2}$

17. If a, b, c are in G.P. and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, then xyz are in:

(A) AP (B) GP (C) HP (D) None of these.

Ans. :

a. AP

Solution:

a, b and c are in G.P.

$\therefore b^2 = ac$

Taking log on both the sides:

$2\log b = \log a + \log c \cdots(i)$

Now, $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$

Taking log on both the sides:

$\frac{\log a}{x} = \frac{\log b}{y} = \frac{\log c}{z} \cdots(ii)$

Now, comparing (i) and (ii):

$\frac{\log a}{x} = \frac{\log a + \log c}{2y} = \frac{\log c}{z}$

$\Rightarrow \frac{\log a}{x} = \frac{\log a + \log c}{2y}$ and $\frac{\log a}{x} = \frac{\log c}{z}$

$\Rightarrow \log a(2y - x) = x\log c$ and $\frac{\log a}{\log c} = \frac{x}{z}$

$\Rightarrow \frac{\log a}{\log c} = \frac{x}{(2y-x)}$ and $\frac{\log a}{\log c} = \frac{x}{z}$

$\Rightarrow \frac{x}{2y-x} = \frac{x}{z}$

$\Rightarrow 2y = x + z$

Thus, x, y and z are in A.P.

18. The two geometric means between the numbers 1 and 64 are:

(A) 1 and 64 (B) 4 and 16 (C) 2 and 16 (D) 8 and 16.

Ans. :

b. 4 and 16

Solution:

Let the two G.M.s between 1 and 64 be G_1 and G_2.

Thus, 1, G_1, G_2 and 64 are in G.P.

$64 = 1 \times r^3$

$\Rightarrow r = \sqrt[3]{64}$

$\Rightarrow r = 4$

$\Rightarrow G_1 = ar = 1 \times 4 = 4$

And, $G_2 = ar^2 = 1 \times 4^2 = 16$

Thus, 4 and 16 are the required G.M.s.

19. If S be the sum, P the product and R be the sum of the reciprocals of n terms of a G.P. then P^2 is equal to:

(A) $\frac{S}{R}$ (B) $\frac{R}{S}$ (C) $\left(\frac{R}{S}\right)^n$ (D) $\left(\frac{S}{R}\right)^n$.

Ans. :

d. $\left(\frac{S}{R}\right)^n$

Solution:

Sum of n terms of the G.P., $S = \frac{a(r^n-1)}{(r-1)}$

Product of n terms of the G.P., $P = a^n r^{\left[\frac{n(n-1)}{2}\right]}$

Sum of the reciprocals of n terms of the G.P., $R = \frac{\left[\frac{1}{r^n}-1\right]}{a\left(\frac{1}{r}-1\right)} = \frac{(r^n-1)}{ar^{(n-1)}(r-1)}$

$\therefore P^2 = \left\{a^2 r^{\frac{2(n-1)}{2}}\right\}^n$

$\Rightarrow P^2 = \left\{\frac{\frac{a(r^n-1)}{(r-1)}}{\frac{(r^n-1)}{ar^{(n-1)(r-1)}}}\right\}^n$

$\Rightarrow P^2 = \left\{\frac{S}{R}\right\}^n$

Let the first term of the G.P. be a and the common ratio be r.

Sum of n terms, $S = \frac{a(r^n-1)}{r-1}$

Product of the G.P., $P = a^n r^{\frac{n(n+1)}{2}}$

Sum of the reciprocals of n terms, $\Rightarrow R = \frac{\left(\frac{1}{r^{n-1}}\right)}{a\left(\frac{1}{r-1}\right)} = \frac{\left(\frac{1-r^n}{r^n}\right)}{a\left(\frac{1}{r-1}\right)}$

$P^2 = \left\{a^2 r^{\frac{(n+1)}{2}}\right\}^n$

$$P^2 = \left\{ \frac{\frac{a(r^n-1)}{r-1}}{\frac{\left(\frac{1-r^n}{r^n}\right)}{a\left(\frac{1-r}{r}\right)}} \right\} = \left\{\frac{S}{R}\right\}^n$$

20. The sum to n terms of the series $\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \ ....\ \text{is}:$

(A) $\sqrt{2n+1}$ (B) $\frac{1}{2}\sqrt{2n+1}$ (C) $\sqrt{2n+1}-1$ (D) $\frac{1}{2}\left\{\sqrt{2n+1}-1\right\}$

Ans. :

d. $\frac{1}{2}\{\sqrt{2n+1}-1\}$

Solution:

Let T_n be the nth term of the given series.

Thus, we have

$$T_n = \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}}$$
$$= \frac{\sqrt{2n+1}-\sqrt{2n-1}}{2}$$

Now,

Let S_n be the sum n terms of the given series.

Thus, we have

$$S_n = \sum_{k=1}^{n} T_k$$
$$= \sum_{k=1}^{n}\left(\frac{\sqrt{2k+1}-\sqrt{2k-1}}{2}\right)$$
$$= \frac{1}{2}\sum_{k=1}^{n}\left(\sqrt{2k+1}-\sqrt{2k-1}\right)$$
$$= \frac{1}{2}\left[\left(\sqrt{3}-\sqrt{1}\right)+\left(\sqrt{5}-\sqrt{3}\right)+\left(\sqrt{7}-\sqrt{5}\right)+\ ...\ +\left(\sqrt{2n+1}-\sqrt{2n-1}\right)\right]$$
$$= \frac{1}{2}\left\{(-1)+\sqrt{2n+1}\right\}$$
$$= \frac{1}{2}\left\{\sqrt{2n+1}-1\right\}$$

21. If $\sum n = 210,$ then $\sum n^2 =$

(A) 2870 (B) 2160 (C) 2970 (D) none of these.

Ans. :

a. 2870

Solution:

Given,

$\sum n = 210$

$\Rightarrow n\left(\frac{n+1}{2}\right) = 210$

$\Rightarrow n^2 + n - 420 = 0$

$\Rightarrow (n - 20)(n + 21) = 0$

$\Rightarrow n = 20\ (\because\ n > 0)$

Now,

$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow \frac{n(n+1)}{2} \times \frac{(2n+1)}{3}$

$\Rightarrow (210) \times \left(\frac{41}{3}\right)$

$\Rightarrow (70) \times (41)$

$\Rightarrow 2870$

22. The consecutive digits of a three digit number are in GP. If the middle digit be increased by 2, then they form an AP. If 792 is subtracted from this, then we get the number constituting of same three digits but in reverse order. Then, number is divisible by:

(A) 7 (B) 49 (C) 19 (D) None of these

Ans. :

a. 7

23. The sum of the series $1^2 + 3^2 + 5^2 + \ldots$ to n terms is:

(A) $\frac{n(n+1)(2n+1)}{2}$ (B) $\frac{n(2n-1)(2n+1)}{3}$ (C) $\frac{(n-1)^2(2n+1)}{6}$ (D) $\frac{(2n+1)^3}{3}$

Ans. :

b. $\frac{n(2n-1)(2n+1)}{3}$

Solution:

Let T_n be the n^{th} term of the given series.

Thus, we have

$T_n = (2n - 1)^2$

$= 4n^2 + 1 - 4n$

Now, let S_n be the sum of n terms of the given series.

Thus, we have

$S_n = \sum_{k=1}^{n} (4k^2 + 1 - 4k)$

$$\Rightarrow S_n = 4\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 1 - 4\sum_{k=1}^{n} k$$
$$\Rightarrow S_n = \frac{4n(n+1)(2n+1)}{6} + n - \frac{4n(n+1)}{2}$$
$$\Rightarrow S_n = \frac{2n(n+1)(2n+1)}{3} + n - 2n(n+1)$$
$$\Rightarrow S_n = n\left[\frac{2(n+1)(2n+1)}{3} + 1 - 2(n+1)\right]$$
$$\Rightarrow S_n = \frac{n}{3}\left[(2n+2)(2n+1) + 3 - 6(n+1)\right]$$
$$\Rightarrow S_n = \frac{n}{3}\left[(4n^2 - 1)\right]$$
$$\Rightarrow S_n = \frac{n(2n-1)(2n+1)}{3}$$

24. The product $(32), (32)^{\frac{1}{6}}(32)^{\frac{1}{36}} \ldots$ to ∞ is equal to:

(A) 64 (B) 16 (C) 32 (D) 0

Ans. :

a. 64

Solution:

$$32 \times 32^{\frac{1}{6}} \times 32^{\frac{1}{36}} \times \cdots\infty$$
$$= 32^{\left(1+\frac{1}{6}+\frac{1}{36}+\cdots\infty\right)}$$
$$= 32^{\left(\frac{1}{1-\frac{1}{6}}\right)} \text{ [∵ it is a G.P.]}$$
$$= 32^{\left(\frac{6}{5}\right)}$$
$$= \left(2^5\right)^{\left(\frac{6}{5}\right)}$$
$$= 2^6$$
$$= 64$$

25. The sum of first three terms of a G.P. is $\frac{21}{2}$ and their product is 27. Which of the following is not a term of the G.P. if the numbers are positive?

(A) 3 (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) 6

Ans. :

b. $\frac{2}{3}$

Solution:

Let three terms be $\frac{a}{r}$, a, × ar.

$$\text{product} = 27 \Rightarrow \left(\frac{a}{r}\right)\left(a\right)\left(a \times r\right) = 27 \Rightarrow a^3 = 27 \Rightarrow a = 3.$$
$$\text{sum} = \frac{21}{2} \Rightarrow \left(\frac{a}{r + a + a \times r}\right) = \frac{21}{2} \Rightarrow a\left(\frac{1}{r+1+1 \times r}\right) = \frac{21}{2}$$

$\Rightarrow \left(\frac{1}{r+1+1\times r}\right) = \left(\frac{\frac{21}{2}}{3}\right) = \frac{7}{2}$

$\Rightarrow \left(r^2 + r + 1 = \right)\left(\frac{7}{2}\right) \Rightarrow r^2 - \left(\frac{5}{2}\right)r + 1 = 0$

$\Rightarrow r = 2 \text{ and } \frac{1}{2}.$

Terms are $\frac{3}{2}$, 3, 3 × 2 i.

e.$\frac{3}{2}$, 3, 6.

26. If 100 times the 100th term of an AP with non-zero common difference equals the 50 times its 50th term, then the 150th term of this AP is:

(A) –150 (B) 150 times its 50th term

(C) 150 (D) zero

Ans. :

d. zero

27. The AM,HMandGMbetween two numbers are $\frac{144}{15}, 15$ and 12, but not necessarily in thisorder. Then, HM, GM and AM respectively are:

(A) $15, 12, \frac{144}{15}$ (B) $\frac{144}{15}, 12, 15$ (C) $15, 12, \frac{144}{15}$ (D) $\frac{144}{15}, 15, 12$

Ans. : $\frac{144}{15}, 12, 15$

28. Choose the correct answer.

If the third term of G.P. is 4, then the product of its first 5 terms is:

(A) 4^3 (B) 4^4 (C) 4^5 (D) None of these.

Ans. :

c. 4^5.

Solution:

Given that:

$T_3 = 4$

$\Rightarrow ar^{3-1} = 4$ $[\because T_n = ar^{n-1}]$

$\Rightarrow ar^2 = 4$

Product of first 5 terms = a. ar. ar^2. ar^3. ar^4

$= a^5r^{10} = (ar^2)^5 = (4)^5$

Hence, the corrrect option is (c).

29. If second term of a G.P. is 2 and the sum of its infinite terms is 8, then its first terms is:

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 2 (D) 4.

Ans. :

d. 4.

Solution:

$a_2 = 2$

$\therefore ar = 2 \cdots$(i)

Also, $S_\infty = 8$

$\Rightarrow \frac{a}{(1-r)} = 8$

$\Rightarrow \frac{a}{\left(1-\frac{2}{a}\right)} = 8$ [Using (i)]

$\Rightarrow a^2 = 8(a-2)$

$\Rightarrow a^2 - 8a + 16 = 0$

$\Rightarrow (a-4)^2 = 0$

$\Rightarrow a = 4$

30. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then, the common ratio of this progression is equal to:

(A) $\frac{1}{2}(1-\sqrt{5})$ (B) $\frac{1}{2}.\sqrt{5}$ (C) $\sqrt{5}$ (D) $\frac{1}{2}(\sqrt{5-1})$

Ans. :

c. $\sqrt{5}$

31. The sum of n terms of two arithmetic progressions are in the ratio (2n + 3) : (7n + 5). Find the ratio of their 9th terms.

(A) 4 : 5 (B) 5 : 4 (C) 9 : 31 (D) 31 : 9

Ans. :

c. 9 : 31

Solution:

xplanation: Let a, a’ be the first terms and d, d’ be the common differences of 2 A.P.’s respectively.

Given, $\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d]} = \frac{2n+3}{7n+5}$

$\Rightarrow \frac{a+(n-1)\frac{d}{2}}{a'+(n-\frac{d'}{2}} = \frac{2n+3}{7n+5}$

If we have to find ratio of 9^{th} terms then

$\frac{(n-1)}{2} = 8 \Rightarrow n = 17$

$\Rightarrow \frac{a+8d}{a'+8d'} = \frac{2\times 17+3}{3\times 17+5} = \frac{34+3}{119+5} = \frac{36}{124} = \frac{9}{31}$.

32. The sum of the series $\frac{2}{3} + \frac{8}{9} + \frac{26}{27} + \frac{80}{81} + \ldots$ to n terms is:

(A) $n-\frac{1}{2}(3^{-n}-1)$ (B) $n-\frac{1}{2}(1-3^{-n})$ (C) $n+\frac{1}{2}(3^n-1)$ (D) $n-\frac{1}{1}(3^n-1)$

Ans. :

b. $n-\frac{1}{2}(1-3^{-n})$

Solution:

Let T_n be the n^{th} term of the given series.

Thus, we have

$T_n = \frac{3^n - 1}{3^n} = 1 - \frac{1}{3^n}$

Now,

Let S_n be the sum of n terms of the given series.

Thus, we have

$S_n = \sum_{k=1}^{n} T_k$

$= \sum_{k=1}^{n} \left[1 - \frac{1}{3^k}\right]$

$= \sum_{k=1}^{n} 1 - \sum_{k=1}^{n} \frac{1}{3^k}$

$= n - \left[\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ .\ .\ .\ .\ + \frac{1}{3^n}\right]$

$= n - \frac{1}{3}\left[\frac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}\right]$

$= n - \frac{1}{2}\left[1 - \left(\frac{1}{3}\right)^n\right]$

$= n - \frac{1}{3}\left[1 - 3^{-n}\right]$

33. Choose the correct answer.

The lengths of three unequal edges of a rectangular solid block are in G.P. If the volume of the block is 216cm^3 and the total surface area is 252cm^2, then the length of the longest edge is:

(A) 12cm (B) 6cm (C) 18cm (D) 3cm

Ans. :

a. 12cm.

Solution:

Let the length, breadth and height of rectangular solid block be $\frac{a}{r}$, a and ar, respectively.

$\therefore$ Volume $= \frac{a}{r} \times a \times ar = 216\text{cm}^3$

$\Rightarrow a^3 = 216 = 6^3 \Rightarrow a = 6$

Also, Surface area $= 2\left(\frac{a}{r}.a + a.ar + \frac{a}{r}.ar\right) = 252$

$\Rightarrow 2a^2\left(\frac{1}{r} + r + 1\right) = 252$

$\Rightarrow 2 \times 36\left(\frac{1+r^2+r}{r}\right) = 252$

$\Rightarrow 2(1 + r^2 + r) = 7r$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow (2r - 1)(r - 2) = 0$

$\therefore\ r = \frac{1}{2}, 2$

For $r = \frac{1}{2}$: Length $= \frac{a}{r} = \frac{6 \times 2}{1} = 12$, Breadth = a = 6

Height $= ar = 6 \times \frac{1}{2} = 3$

For r = 2: Length $= \frac{a}{r} = \frac{6}{2} = 3$, Breadth = a = 6

Height = ar = 6 × 2 = 12

34. The value of $9^{\frac{1}{3}}.9^{\frac{1}{9}}.9^{\frac{1}{27}}\ldots$to ∞, is:

(A) 1 (B) 3 (C) 9 (D) None of these.

Ans. :

b. 3

Solution:

$9^{\frac{1}{3}} \times 9^{\frac{1}{9}} \times 9^{\frac{1}{27}} \times \ldots\infty$

$= 9^{\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\infty\right)}$

Here, it is a G.P. with $a = \frac{1}{3}$ and $r = \frac{1}{3}$.

$\therefore 9^{\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)}$

$= 9^{\left(\frac{1}{2}\right)} = 3$

35. Find the sum to n terms of the series whose nth term is n (n-2).

(A) $\frac{n(n-1)(2n+4)}{6}$ (B) $\frac{n(n-1)(2n-5)}{6}$ (C) $\frac{(n-1)(2n-5)}{3}$ (D) $\frac{n(n-1)(2n-5)}{3}$

Ans. :

b. $\frac{n(n-1)(2n-5)}{6}$

Solution:

Given, n^{th} term is n(n-2)So, a_k = k(k-2)

Taking summation from k=1 to k=n on both sides, we get

$\sum_{i}^{n} = 0\ a_k = \sum_{i}^{n} = 0\ k^2 - 2\sum_{i}^{n} = 0\ k = \frac{n(n+1)(2n+1}{6} - 2\frac{n(n+1)}{2} = \frac{n(n+1)(2n-5)}{6}$.

36. The value of $\sum_{r=1}^{n}\left\{(2r-1)a + \frac{1}{b^r}\right\}$ is equal to:

(A) $an^2 + \frac{b^{n-1}-1}{b^{n-1}(b-1)}$ (B) $an^2 + \frac{b^n-1}{b^n(b-1)}$ (C) $an^3 + \frac{b^{n-1}-1}{b^n(b-1)}$ (D) none of these.

Ans. :

b. $an^2 + \frac{b^n-1}{b^n(b-1)}$

Solution:

We have,

$$\sum_{r=1}^{n} \left\{(2r-1)a + \frac{1}{b^r}\right\}$$

$$= \sum_{r=1}^{n} \left\{2ra - a + \frac{1}{b^r}\right\}$$

$$= \sum_{r=1}^{n} 2ar - \sum_{r=1}^{n} a + \sum_{r=1}^{n} \frac{1}{b^r}$$

$$= an(n+1) - an + \frac{(1-b^n)}{(1-b)b^n}$$

$$= an^2 + \frac{b^n-1}{b^n(b-1)}$$

37. Given that x > 0, the sum $\sum_{n=1}^{\infty} \left(\frac{x}{x+1}\right)^{n-1}$ equals:

(A) x (B) $x+1$ (C) $\frac{x}{2x+1}$ (D) $\frac{x+1}{2x+1}$

Ans. :

b. $x+1$

Solution:

$$\sum_{x+1}^{\infty} \left(\frac{x}{x+1}\right)^{(n-1)}$$

$$= 1 + \left(\frac{x}{x+1}\right) + \left(\frac{x}{x+1}\right)^2 + \left(\frac{x}{x+1}\right)^3 + \left(\frac{x}{x+1}\right)^4 + \ldots\infty$$

$$= \frac{1}{1-\left(\frac{x}{x+1}\right)} \left[\because \text{ it is a G.P. with } a = 1 \text{ and } r = \left(\frac{x}{x+1}\right)\right]$$

$$= \frac{(x+1)}{(x+1-x)}$$

$$= \frac{(x+1)}{1} = (x+1)$$

38. Choose the correct answer.

If in an A.P., $S_n = qn^2$ and $S_m = qm^2$, where S_r denotes the sum of r terms of the AP, then S_q equals:

(A) $\frac{q^3}{2}$ (B) mnq (C) q^3 (D) $(m+n)q^2$

Ans. :

c. q^3.

Solution:

Given,

$S_n = qn^2$ and $S_m = qm^2$

$\therefore$ $S_1 = q$, $S_2 = 4q$, $S_3 = 9q$ and $S_4 = 16q$

Now, $t_1 = q$

$\therefore$ $t_2 = S_2 - S_1 = 4q - q = 3q$

$t_3 = S_3 - S_2 = 9q - 4q = 5q$

$t_4 = S_4 - S_3 = 16q - 9q = 7q$

So, the A.P. is: q, 3q, 5q, 7q, ...

Thus, first term is q and common difference is 3q - q = 2q.

$\therefore\ S_q = \frac{q}{2}[2 \times q + (q-1)2q] = \frac{q}{2} \times [2q + 2q^2 - 2q]$

$= \frac{q}{2} \times 2q^2 = q^3$

39. Jairam purchased a house in Rs. 15000 and paid Rs. 5000 at once. Rest money he promised to pay in annual instalment of Rs. 1000 with 10% per annum interest. How much money is to be paid by Jairam?

(A) Rs. 21555 (B) Rs. 20475 (C) Rs. 20500 (D) Rs. 20700

Ans. :

c. Rs. 20500

40. Let x be the A.M. and y, z be two G.M.s between two positive numbers. Then, $\frac{y^3+z^3}{xyz}$ is equal to:

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) None of these.

Ans. :

b. 2

Solution:

Let the two numbers be a and b.

a, x and b are in A.P.

$\therefore 2x = a + b \cdots(i)$

Also, a, y, z and b are in G.P.

$\therefore \frac{y}{a} = \frac{z}{y} = \frac{b}{z}$

$\Rightarrow y^2 = az, yz = ab, z^2 = by \cdots(ii)$

Now, $\frac{y^3+z^3}{xyz}$

$= \frac{y^2}{xz} + \frac{z^2}{xy}$

$= \frac{1}{x}\left(\frac{y^2}{z} + \frac{z^2}{y}\right)$

$= \frac{1}{x}\left(\frac{az}{z} + \frac{by}{y}\right)$ [Using (ii)]
$= \frac{1}{x}(a + b)$
$= \frac{2}{(a+b)}(a + b)$ [Using (i)]
$= 2$

41. Find the sum $1^3+2^3+3^3+\ldots\ldots\ldots\ldots..+8^3$.

(A) 1225 (B) 1184 (C) 1475 (D) 1296

Ans. :

d. 1296

Solution:

We know, sum of cubes of first n terms is given by
$\left(\frac{n(n+1)}{2}\right)^2$.

$\text{Here, } n = 8 \text{ so, sum } = \left(\frac{8\times 9}{2}\right)^2 + 1296.$

42. Find the sum to 6 terms of each of the series 2*3+4*6+6*11+8*18+...................

(A) 784 (B) 882 (C) 928 (D) 966

Ans. :

d. 966

Solution:

General term of above series is $a_k = 2k*(k^2+2) = 2k^3+4k$

Taking summation from k=1 to k=n on both sides, we get

$\sum_{i}^{n} = 0\ a_k = 2\sum_{i}^{n} = 0\ k^3 + 4\sum_{i}^{n} = 0\ k = 2\left(\frac{(n(n+1)}{2}\right)^2 + 4\frac{n(n+1)}{2}$
$= n^2\frac{(n+1)}{2} + 2n(n+1)$
$= \frac{36\times 49}{2} + 2 \times 6 \times 7 = 966.$

43. Choose the correct answer.
If x, 2y and 3z are in A.P. where the distinct numbers x, y and z are in G.P., then the common ratio of the G.P. is:

(A) 3 (B) $\frac{1}{3}$ (C) 2 (D) $\frac{1}{2}$

Ans. :

b. $\frac{1}{2}$.

Solution:

Since, x, 2y and 3z are in A. P., we get
$2y = \frac{x+3z}{2}$
$\Rightarrow 4y = x + 3z$
Also, x, y and z are ibn G.P.

Therefore, $y = xr$ and $z = xr^2$.
Where 'r' is the common ratio.

$\therefore\ 4xr = x + 3xr^2$ [Using (1)]

$\Rightarrow 4r = 1 + 3r^2$

$\Rightarrow 3r^2 - 4r + 1 = 0$

$\Rightarrow (3r - 1)(r - 1) = .0$

$\Rightarrow r = \frac{1}{3}$

(For r = 1; x, y, z are not distinct)

44. The ratio of the A.M. and G.M. of two positive numbers a and b is 5 : 3. Find the ratio of a to b.

(A) 9 : 1 (B) 3 : 5 (C) 1 : 9 (D) 3 : 1

Ans. :

a. 9 : 1

Soultion:

$$\frac{\textbf{(A.M.)}}{\textbf{(G.M.)}} = \frac{\mathbf{5}}{\mathbf{3}}$$

$$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{3}$$

Applying componendo and dividendo rule, we get

$$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{8}{2}$$

$$\Rightarrow \left(\frac{\sqrt{a+b}}{\sqrt{a-b}}\right)^2 = 4$$

$$\Rightarrow \left(\frac{\sqrt{a+b}}{\sqrt{a-b}}\right)^1 = 2$$

$$\left(\frac{\sqrt{a}}{\sqrt{b}}\right)^1 = 3$$

Again applying componendo and dividendo rule, we get

$$\frac{a}{b} = 3\left(\frac{3}{1}\right)^2 = 9.\ \text{so, } a : b = 9 : 1$$

45. Find the sum of series $1^3 + 3^3 + 5^3 +$...............................$+ 11^3$.

(A) 2556 (B) 5248 (C) 6589 (D) 9874

Ans. :

a. 2556

Solution:

$1^3 + 3^3 + 5^3 +$...............................$+ 11^3$

$= (1^3 + 2^3 + 3^3 + ... + 11^3) - (2^3 + 4^3 + 6^3 + 8^3 + 10^3)$

$= (1^3 + 2^3 + 3^3 + ... 11^3) - 2^3(1^3 + 23 + 3^3 + 4^3 + 5^3)$

$\left(\frac{11\times12}{2}\right)^2-8\left(\frac{5\times6}{2}\right)^2$

$=66^2-8\times15^2$

$4356-1800=2556.$

46. Let S be the sum, P be the product and R be the sum of the reciprocals of 3 terms of a G.P. then $P^2R^3 : S^3$ is equal to:

(A) 1 : 1

(B) (common ratio)^n : 1

(C) (First term)^2(common ratio)^2

(D) None of these.

Ans. :

a. 1 : 1

Solution:

Let the three terms of the G.P. be $\frac{a}{r}$, a, ar. Then

$S=\frac{a}{r}+a+ar$

$=a\left(\frac{1}{r}+1+r\right)$

$=a\left(\frac{1+r+r^2}{r}\right)$

$=\frac{a(r^2+r+1)}{r}$

Also,

$P=\frac{a}{r}\times a\times ar=a^3$

And,

$R=\frac{r}{a}+\frac{1}{a}+\frac{1}{ar}$

$=\frac{1}{a}\left(\frac{r^2+r+1}{r}\right)$

Now,

$\frac{P^2R^2}{S^3}=\frac{(a^3)^2\times\left[\frac{1}{a}\left(\frac{r^2+r+1}{r}\right)\right]^3}{\left[a\left(\frac{r^2+r+1}{r}\right)\right]^3}$

$=\frac{a^6\times\frac{1}{a^3}\left(\frac{r^2+r+1}{r}\right)^3}{a^3\left(\frac{r^2+r+1}{r}\right)^3}$

$=\frac{1}{1}$

So, the ratio is 1 : 1.

Hence, the correct alternative is option (a).

47. If in an A.P., first term is 20, common difference is 2 and nth term is 42, then find n.

(A) 10 (B) 11 (C) 12 (D) 14

Ans. :

c. 12

Solution:

We know, a = 20, d = 2, a_n = 42.

a + (n - 1) d = 42 20 ⇒ + 2(n - 1) = 42

⇒ 2 (n - 1) = 42 - 20 = 22 ⇒ n - 1 = 11 ⇒ n = 12.

48. 150 workers were engaged to finish a piece of work in a certain number of days. 4 workers dropped the second day, 4 more workers dropped the third day and so on. It takes eight more days to finish the work now. The number of days in which the work was completed is:

(A) 15 (B) 20 (C) 25 (D) 30

Ans. :

c. 25

49. Find the sum of series $6^3 + 7^3 + \ldots + 20^3$.

(A) 43875 (B) 83775 (C) 43775 (D) 43975

Ans. :

a. 43875

Solution:

$6^3 + 7^3 + \ldots + 20^3$

$= (1^3 + 2^3 + 3^3 + \ldots + 20^3) - (1^3 + 2^3 + 3^3 + 4^3 + 5^3)$

$= \left(\frac{20 \times 21}{2}\right)^2 - \left(\frac{5 \times 6}{2}\right)^2$

$= (210)^2 - (15)^2$

$= 225 \times 195$

$= 43875$

50. Find the sum of cubes of first n terms

(A) $\frac{n(n+1)}{2}$ (B) $\left(\frac{n(n+1)}{2}\right)^3$ (C) $\frac{n(n+1)(2n+1}{6}$ (D) $\left(\frac{n(n+1)}{2}\right)$

Ans. :

c. $\frac{n(n+1)(2n+1}{6}$

Solution:

Sum of cubes of first n terms = $1^3+2^3+3^3+\ldots+n^3$

$(k + 1)^4 - k^4 = 4k^3 + 6k^2 + 4k + 1$.

On substituting k = 1, 2, 3, ……, n and adding we get,

$4n^3 + n^4 + 6n^2 + 4n = 4$

$n = 4\sum_{i}^{n} = 0\ k^3 + 6\sum_{i}^{n} = 0\ k^2 + 4\sum_{i}^{n} = 0\ k + n$

$4n^3 + n^4 + 6n^2 + 4n = 4$

$$= 4\sum_{i}^{n} = 0\ k^{3} + 6\frac{(n(n+1)(2n+1))}{6} + 4\frac{n(n+1)}{2} + n\sum_{i}^{n} = 0\ k_i^{n} = 0\ k^{3} = \left(\frac{n(n+1)}{2}\right)^{2}.$$

51. The value of $\sum_{r=1}^{n} \log\left(\frac{a^r}{b^{r-1}}\right)$ is

(A) $\frac{n}{2}\log\left(\frac{a^n}{b^n}\right)$ (B) $\frac{n}{2}\log\left(\frac{a^{n+1}}{b^n}\right)$ (C) $\frac{n}{2}\log\left(\frac{a^{n+1}}{b^{n-1}}\right)$ (D) $\frac{n}{2}\log\left(\frac{a^{n+1}}{b^{n+1}}\right)$

Ans. : c

(c) The given series is

$$\log a + \log\left(\frac{a^2}{b}\right) + \log\left(\frac{a^3}{b^2}\right) + \log\left(\frac{a^4}{b^3}\right) + \ldots\ldots + \log\left(\frac{a^n}{b^{n-1}}\right)$$

This is an $A.P.$ with first term $\log a$

and the common difference $\log\left(\frac{a^2}{b}\right) - \log a = \log\left(\frac{a}{b}\right)$

Therefore the sum of n terms is

$$\frac{n}{2}\left[\log a + \log\left(\frac{a^n}{b^{n-1}}\right)\right] = \frac{n}{2}\log\left(\frac{a^{n+1}}{b^{n-1}}\right).$$

Trick : Check for $n = 1,\ 2$.

52. The sum of all two digit numbers which, when divided by 4, yield unity as a remainder is

(A) 1190 (B) 1197 (C) 1210 (D) None of these

Ans. : c

(c) The given numbers are $13, 17, \ldots\ldots 97$.

This is an AP with first term 13 and common difference 4.

Let the number of terms be n.

Then $97 = 13 + (n-1)4$

$\Rightarrow\ 4n = 88$

$\Rightarrow\ n = 22$

Therefore the sum of the numbers

$= \frac{22}{2}[13 + 97] = 11(110) = 1210.$

53. The sum of the integers from 1 to 100 which are not divisible by 3 or 5 is

(A) 2489 (B) 4735 (C) 2317 (D) 2632

Ans. : d

(d) Let $S = 1 + 2 + 3 + \ldots\ldots\ldots\ldots + 100$

$= \frac{100}{2}(1 + 100) = 50(101) = 5050$

Let $S_1 = 3 + 6 + 9 + 12 + \ldots\ldots\ldots + 99$

$=3(1 + 2 + 3 + 4 + \ldots\ldots\ldots + 33)$

$=3.\frac{33}{2}(1 + 33) = 99 \times 17 = 1683$

Let $S_2 = 5 + 10 + 15 + \ldots\ldots\ldots + 100$

$= 5(1 + 2 + 3 + \ldots\ldots\ldots + 20)$

$= 5.\frac{20}{2}(1 + 20) = 50 \times 21 = 1050$

Let $S_3 = 15 + 30 + 45 + \ldots\ldots\ldots + 90$

$= 15(1 + 2 + 3 + \ldots\ldots\ldots + 6)$

$= 15.\frac{6}{2}(1 + 6) = 45 \times 7 = 315$

Required sum $=S - S_1 - S_2 + S_3$

$= 5050 - 1683 - 1050 + 315 = 2632.$

54. If twice the 11^{th} term of an $A.P.$ is equal to 7 times of its 21^{st} term, then its 25^{th} term is equal to

(A) 24 (B) 120 (C) 0 (D) None of these

Ans. : c

(c) Let the first term of $A.P.$ is a and common difference is d.

11^{th} term of $A.P. = a + 10d$

21^{st} term of $A.P. = a + 20d$

$2(a + 10d) = 7(a + 20d)$

$==> 2a + 20d = 7a + 140d$

$5a + 120d = 0$

$==> a + 24d = 0$

Hence 25^{th} term is 0.

55. If $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$ are in $H.P.$, then

(A) $a^2b,\ c^2a,\ b^2c$ are in $A.P.$

(B) $a^2b,\ b^2c,\ c^2a$ are in $H.P.$

(C) $a^2b,\ b^2c,\ c^2a$ are in $G.P.$

(D) None of these

Ans. : a

(a) $\frac{b}{a}, \frac{c}{b}, \frac{a}{c}$ are in $A.P.$

$==> \frac{2c}{b} = \frac{b}{a} + \frac{a}{c}$

$\Rightarrow \frac{2c}{b} = \frac{bc+a^2}{ac}$

$==> 2ac^2 = b^2c + ba^2$

$\therefore\ a^2b,\ c^2a$ and b^2c are in $A.P.$

56. If $a^2,\ b^2,\ c^2$ be in $A.P.$, then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ will be in

(A) $A.P.$ (B) $G.P.$ (C) $H.P.$ (D) None of these

Ans. : a

(a) Since $a^2,\ b^2,\ c^2$ be in $A.P.$

Then $b^2 - a^2 = c^2 - b^2$

$\Rightarrow\ (b-a)(b+a) = (c-b)(c+b)$

==> $\frac{b-a}{c+b} = \frac{c-b}{b+a}$

==> $\frac{(b-a)(a+b+c)}{(c+a)(b+c)} = \frac{(c-b)(a+b+c)}{(a+b)(c+a)}$

$\Rightarrow\ \frac{b^2+bc-ac-a^2}{(c+a)(b+c)} = \frac{c^2+ac-ab-b^2}{(a+b)(c+a)}$

$\Rightarrow\ \frac{b}{c+a} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{c+a}$

Hence $\frac{a}{b+c},\ \frac{b}{c+a},\ \frac{c}{a+b}$ be in $A.P.$

57. If the angles of a quadrilateral are in $A.P.$ whose common difference is 10^o, then the angles of the quadrilateral are

(A) $65^o, 85^o, 95^o, 105^o$ (B) $75^o, 85^o, 95^o, 105^o$

(C) $65^o, 75^o, 85^o, 95^o$ (D) $65^o, 95^o, 105^o, 115^o$

Ans. : b

(b) Suppose that $\angle A = x^0$, then $\angle B = x + 10^o$,

$\angle C = x + 20^o$and $\angle D = x + 30^o$

So, we know that $\angle A + \angle B + \angle C + \angle D = 2\pi$

Putting these values, we get

$(x^o) + (x^o + 10^o) + (x^o + 20^o) + (x^o + 30^o) = 360^o$

$\Rightarrow x = 75^o$

Hence the angles of the quadrilateral are $75^o,\ 85^o,\ 95^o,\ 105^o$.

Trick : In these type of questions, students should satisfy the conditions through options.

Here (b) satisfies both the conditions

$i.e.$ angles are in $A.P.$ with common difference 10^o and sum of angles is 360^o.

58. Jairam purchased a house in Rs. 15000 and paid Rs. 5000 at once. Rest money he promised to pay in annual installment of Rs. 1000 with 10% per annum interest. How much money is to be paid by Jairam Rs. ....

(A) 21555 (B) 20475 (C) 20500 (D) 20700

Ans. : c

(c) It will take 10 years for Jairam to pay off Rs. 10000 in 10 yearly installments.

He pays 10% annual interest on remaining amount

$\therefore$ Money given in first year

$= 1000 + \frac{10000 \times 10}{100} = \text{Rs.}2000$

Money given in second year $= 1000 +$ interest of$(10000 - 1000)$ with interest rate 10% per annum $= 1000 + \frac{9000 \times 10}{100} = \text{Rs. } 1900$

Money paid in third year = Rs. 1800 etc.

So money given by Jairam in 10 years will be Rs. 2000, Rs. 1900, Rs. 1800, Rs. 1700,....,

which is in arithmetic progression, whose first term $a = 2000$and $d = -100$

Total money given in 10 years = sum of 10 terms of arithmetic progression

$= \frac{10}{2}[2(2000) + (10 - 1)(-100)]$= Rs. 15500

Therefore, total money given by Jairam

$= 5000 + 15500 = \text{Rs. } 20500.$

59. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in $A.P.$, then their common difference will be

(A) ± 1 (B) ± 2 (C) ± 3 (D) ± 4

Ans. : c

(c) Let $a - d, a, a + d$ be the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$

Then $(a - d) + a + (a + d) = 12$ and $(a - d)\,a\,(a + d) = 28$

==> $3a = 12$ and $a(a^2 - d^2) = 28$

==> $a = 4$ and $a(a^2 - d^2) = 28$

==> $16 - d^2 = 7$

$\Rightarrow d = \pm 3.$

60. The $A.M.$ of a 50 set of numbers is 38. If two numbers of the set, namely 55 and 45 are discarded, the $A.M.$ of the remaining set of numbers is

(A) 38.5 (B) 37.5 (C) 36.5 (D) 36

Ans. : b

(b) Given, $\frac{\Sigma x_i}{50} = 38, \quad \therefore \Sigma x_i = 1900$

New value of $\Sigma x_i = 1900 - 55 - 45 = 1800, n = 48$

New mean $= \frac{1800}{48} = 37.5.$

61. 150 workers were engaged to finish a piece of work in a certain number of days. 4 workers dropped the second day, 4 more workers dropped the third day and so on. It takes eight more days to finish the work now. The number of days in which the work was completed is

(A) 15 (B) 20 (C) 25 (D) 30

Ans. : c

(c) Let the number of days be n.

Hence a worker can do $\left(\frac{1}{150n}\right)^{th}$ part of the work in a day.

Accordingly,

$[150 + 146 + 142 + \ldots\ldots. + \text{upto } (n+8) \text{ terms}] \times \frac{1}{150n} = 1$

$\Rightarrow n = 17$

Therefore number of total days in completion $= 17 + 8 = 25.$

62. Given that n A.M.'s are inserted between two sets of numbers $a,\ 2b$and $2a,\ b$, where $a,\ b \in R$. Suppose further that m^{th} mean between these sets of numbers is same, then the ratio $a : b$ equals

(A) $n - m + 1 : m$ (B) $n - m + 1 : n$ (C) $n : n - m + 1$ (D) $m : n - m + 1$

Ans. : d

(d) m^{th} mean between $a,\ 2b$ is $a + \frac{m(2b-a)}{n+1}$ ......(i)

and m^{th} mean between $2a,\ b$ is $2a + \frac{m(b-2a)}{n+1}$......(ii)

Accordingly, $a + \frac{m(2b-a)}{n+1} = 2a + \frac{m(b-2a)}{n+1}$

$\Rightarrow\ m(2b - a) = a(n+1) + m(b - 2a)$

$\Rightarrow\ a(n - m + 1) = bm$

$\Rightarrow\ \frac{a}{b} = \frac{m}{n-m+1}.$

63. If $\alpha,\ \beta,\ \gamma$ are the geometric means between $ca,\ ab;\ ab,\ bc;\ bc,\ ca$ respectively where $a,\ b,\ c$ are in A.P., then $\alpha^2,\ \beta^2,\ \gamma^2$ are in

(A) $A.P.$ (B) $H.P.$

(C) $G.P.$ (D) None of the above

Ans. : a

(a) By hypothesis, $\alpha^2 = a^2bc,\ \beta^2 = b^2ca,\ \gamma^2 = c^2ab$ and $2b = a + c$.

.Hence $2^{n-1} > 100$ are in A.P.

64. Maximum value of sum of arithmetic progression $50, 48, 46, 44. \ldots\ldots.$ is :-

(A) 325 (B) 648 (C) 652 (D) 650

Ans. : d

For maximum sum $\Rightarrow \mathrm{T_n} = 0$

$a + (n-1)d = 0$

$\Rightarrow 50 + (n-1)(-2) = 0 \Rightarrow n = 26$

So $S_{26} = \frac{26}{2}[2 \times 50 + 25 \times (-2)] = 650$

*** Given section consists of questions of 2 marks each.** **[34]**

65. A manufacturer reckons that the value of a machine, which cost him ₹ 15625 will depreciate each year by 20%. Find the estimated value at the end of 5 years.

Ans. : Present value of the machine = ₹ 15625

Rate of depreciation = 20%

After 1 year value of machine = 15625 - 15625 $\times \frac{20}{100}$ = 15625 – 3125 = ₹ 12500

After 2 year value of machine = 12500 - 12500 $\times \frac{20}{100}$ = 12500 – 2500 = ₹ 10000

After 3 year value of machine = 10000 - 10000 $\times \frac{20}{100}$ = 10000 – 2000 = ₹ 8000

∴ Sequence of values of machine after depreciation is 12500, 10000, 8000, ... is a G.P.

Here a = 12500, r = $\frac{10000}{12500} = \frac{4}{5}$

∴ $a_5 = ar^4 = 12500 \times \left(\frac{4}{5}\right)^4 = 12500 \times \frac{256}{625}$ = ₹5120

Therefore, the value of machine at the end of 5 years is ₹ 5120

66. Find the indicated terms of the sequence, whose nth term is $a_n = \frac{n(n-2)}{n+3}$; a_{20}

Ans. : Given: $a_n = \frac{n(n-2)}{n+3}$

$\therefore a_{20} = \frac{20(20-2)}{20+3} = \frac{20 \times 18}{23} = \frac{360}{23}$

Therefore, 20^{th} term is $\frac{360}{23}$.

67. Write the first five terms of the sequence whose n^{th} term is $a_n = n(n + 2)$

Ans. : Given: $a_n = n(n + 2)$

Putting n = 1, 2, 3, 4 and 5, we get,

$a_1 = 1(1 + 2) = 1 \times 3 = 3$

$a_2 = 2(2 + 2) = 2 \times 4 = 8$

$a_3 = 3(3 + 2) = 3 \times 5 = 15$

$a_4 = 4(4 + 2) = 4 \times 6 = 24$

$a_5 = 5(5 + 2) = 5 \times 7 = 35$

Therefore, the first five terms are 3, 8, 15, 24 and 35.

68. If a, b, c are in G.P., Prove that loga, logb, logc are in A.P.

Ans. : Here, a, b, c are in G.P.

$b^2 = ac \cdots (i)$

Now, $2\log b = \log b^2$

$= \log ac$

$2\log b = \log a + \log c$

$\log b - \log a = \log c - \log b$

$\Rightarrow \log a, \log b \log c,$ are in A.P.

69. If a is the G.M. of 2 and $\frac{1}{4}$, find a.

Ans. : a is the G.M. between 2 and $\frac{1}{4}$,

Then,

$a = \sqrt{2 \times \frac{1}{4}}$

$a = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

70. If a, b, c are in G.P., Prove that $\frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m}$ are in A.P.

Ans. : Here, a, b, c are in G.P., so

$b^2 = ac$

Now, $\frac{2}{\log_b m} = 2\log_m b$

$= \log_m b^2$

$= \log_m ac$

$= \log_m a + \log_m c$

$\frac{2}{\log_b m} = \frac{1}{\log_a m} + \frac{1}{\log_c m}$

$\Rightarrow \frac{1}{\log_b m} - \frac{1}{\log_a m} = \frac{1}{\log_c m} - \frac{1}{\log_b m}$

$\Rightarrow \frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m}$ are in A.P.

71. Find the sum of the following series to infinit:

$10 - 9 + 8.1 - 7.29 + \ ...\infty$

Ans. : This infinity G.P. has first term a = 10 and common ratio $r = -\frac{9}{10} = -0.9$

Thus the sum of the infinity G.P. will be:

$10 - 9 + 8.9 - 7.29 + \ ...\infty = \frac{a}{1-r}$ [Since $|r| < 1$]

$= \frac{10}{1-(-0.9)}$

$= \frac{10}{1.9}$

$= \frac{100}{19}$

$= 5.263$

72. Find:

The 8th term of the G.P. 0.3, 0.06, 0.012, ...

Ans. : Here,

First term, a = 0.3

Common ratio, $r = \frac{a_2}{a_1} = \frac{0.06}{0.3} = 0.2$

$\therefore$ 8^{th} term = a8 = $ar^{(8-1)}$ = 0.3 $(0.2)^7$

Thus, the 8th term of the given GP is 0.3 $(0.2)^7$.

73. Find the sum of the following series to infinity:

$$\frac{2}{5}+\frac{3}{5^2}+\frac{2}{5^3}+\frac{3}{5^4}+\ ...\infty$$

Ans. : $S_\infty = \frac{2}{5}+\frac{3}{5^2}+\frac{2}{5^3}+\frac{3}{5^4}+\ ...$

$= \left(\frac{2}{5}+\frac{3}{5^3}+\ ...\right)+\left(\frac{3}{5^2}+\frac{3}{5^4}+\ ...\right)$

$S_\infty = S'_\infty + S''_\infty$

For

$S'_\infty = \frac{a}{1-r}$

$= \frac{\frac{2}{5}}{1-\frac{1}{25}}$

$= \frac{1}{2}\times\frac{25}{24}$

$S'_\infty = \frac{5}{12}$

$S''_\infty = \frac{\frac{3}{25}}{1-\frac{1}{25}}$

$= \frac{3}{25}\times\frac{25}{24}$

$= \frac{3}{24}$

$S_\infty = S'_\infty + S''_\infty$

$= \frac{5}{12}+\frac{3}{24}$

$= \frac{13}{24}$

$S_\infty = \frac{13}{24}$

74. Find:

The 12^{th} term of the G.P. $\frac{1}{a^3x^3}$, ax, a^5x^5...

Ans. : 12^{th} term of the G.P. $\frac{1}{a^3x^3}$, ax, a^5x^5...

$a = \frac{1}{a^3x^3}$

$r = \frac{t_n}{t_{n-1}} = \frac{t_2}{t_1} = \frac{ax}{\frac{1}{a^3x^3}} = a^4x^4$

$t_n = ar^{n-1}$

$t_{12} = ar^{11}$

$= \left(\frac{1}{a^3x^3}\right)(a^4x^4)^{11}$

$= (ax)^{41}$

75. Find:

The ninth term of the G.P. 1, 4, 16, 64, ...

Ans. : 9^{th} term of G.P. 1, 4, 16, 64, ...

$t_1 = 1 = a$

$t_2 = 4$

Because it is G.P.

$\frac{t_2}{t_1} = \text{common ratio} = r$

$r = \frac{4}{1} = 4$

$t_n = ar^{n-1}$

$t_9 = ar^8 = 1(4)^8 = 4^8$

76. Find the sum of the following series to infinity:

$$1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \frac{1}{3^4} + \ ....\infty$$

Ans. : $S_\infty = 1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \ ...$

$\Rightarrow a = 1, r = \ -\frac{1}{3}$

$S_\infty = \frac{a}{1-r}$

$= \frac{1}{1+\frac{1}{3}}$

$S_\infty = \frac{3}{4}$

77. How many terms of the series 2 + 6 + 18 + ... must be make the sum equal to 728₹

Ans. : 2 + 6 + 18 + ...

$S_n = 728$

Now,

$S_n = \frac{a(1-r^n)}{1-r}$

$a = 2, r = \frac{6}{2} = 3$

$728 = \frac{2(3^n-1)}{3-1}$

$728 = \frac{2(3^n-1)}{2} = (3^n - 1)$

$728 + 1 = 3^n$

$729 = 3^n$

$(3)^6 = 3n$

$\Rightarrow n = 6$

78. The sum of three numbers which are consecutive terms of an A.P. is 21. If the second number is reduced by 1 and the third is increased by 1, we obtain three consecutive terms of a G.P. Find the numbers.

Ans. : Let three numbers in A.P. be a - d, a + d

Here, a - d + a + a + d =21

3a = 21

a = 7

And,

(7 - d), (7 - 1), (7 + d) + 1 are in G.P.

(7 - d), 6, (8 + d) are in G.P.

$(6)^2 = (7 - d)(8 + d)$

$36 = 56 + 7d - 8d - d^2$

$d^2 + d - 20 = 0$

(d + 5) (d - 4) = 0

d = 4, -5

So, Numbers are 3, 7, 11 or 12, 7, 2.

79. Find the sum of the following geometric progrssions:

1, 3, 9, 27, ... to 8 terms

Ans. : 1, 3, 9, 27, ... to 8 terms

$a = 1, r = \frac{3}{1} = 3, n = 8$

$S_n = a\frac{(r^{n-1})}{r-1}$

$S_8 = 1\frac{(3^8-1)}{3-1} = 3280$

80. The sum of interior angles of a triangle is 180°. Show that the sum of the interior angles of polygons with 3, 4, 5, 6,... sides form an arithmetic progression. Find the sum of the interior angles for a 21 sided polygon.

Ans. : We know that, sum of interior angles of a polygon of side n is (n - 2) × 180°.

Let $t_n = (n - 2) \times 180°$

Since, t_n is linear in n, it is n^{th} term of some A.P.

$t_3 = a = (3 - 2) \times 180° = 180°$

Common difference d = 180°

Sum of the interior angles for a 21 sided polygon is:

$t_{21} = (21 - 2) \times 180° = 3420°$

81. The first term of an A.P. is a and the sum of the first p terms is zero, show that the sum of its next q term is $\frac{-a(p+q)q}{p-1}$.

[**Hint:** Required sum = $S_{p+q} - S_p$]

Ans. : Let the common differeence of the given A.P be d.

Given that $S_p = 0$

$\Rightarrow \frac{p}{2}[2a + (p-1)d] = 0$

$\Rightarrow 2a + (p-1)d = 0$

$\Rightarrow d = \frac{-2a}{p-1}$

Now, sum of next q terms,

$= S_{p+q} - S_p = S_{p+q} - 0$

$= \frac{p+q}{2}[2a + (p+q-1)d]$

$= \frac{p+q}{2}[2a + (p-1)d + qd]$

$= \frac{p+q}{2}\left[0 + \frac{q-2a}{p-1}\right]$

$= \frac{-a(p+q)q}{p-1}$

*** Given section consists of questions of 3 marks each.** **[129]**

82. The sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2 respectively. Find the last term and the number of terms.

Ans. : Given: a = 15, r = 2 and $S_n = 315$

$\therefore S_n = \frac{a\left(r^n - 1\right)}{r-1}$

$\Rightarrow 315 = \frac{5\left(2^n - 1\right)}{2-1}$

$\Rightarrow \frac{315}{5} = 2^n - 1$

$\Rightarrow 2^{n-1} = 63$

$\Rightarrow 2^n = 64 = 2^6$

$\Rightarrow n = 6$

$\therefore a_6 = ar^{6-1} = 5 \times 2^5 = 5 \times 32 = 160$

Hence the number of terms=6 and the last term =160

83. If f is a function satisfying f (x+y) = f (x) f (y) for all x, y $\in$ N such that f (1) = 3 and $\sum_{x=1}^{n} f(x) = 120$ find the value of n.

Ans. : f (1) = 3

$f(1 + 2) = f(1)\ f(2) = 3 \times 9 = 27$

$f(1 + 3) = f(1)\ f(3) = 3 \times 27 = 81$

L.H.S.

$= f(1) + f(2) + f(3) + \ldots\ldots + f(n)$

$= 3 + 9 + 27 + 81 + \ldots + n$ terms

$= \frac{3(3^n - 1)}{3 - 1} = \frac{3}{2}(3^n - 1)$

$= f(1) + f(2) + f(3) + \text{----} + f(n)$

$= 3 + 9 + 27 + 1 + \text{-----} + n$ term s

$$= \frac{3\left(3^n - 1\right)}{3 - 1} = \frac{3}{2}\left(3^n - 1\right)$$

ATQ

$$\frac{3}{2}\left(3^n - 1\right) = 120$$

$3^n - 1 = 80$

$3^n = 81$

$n = 4$

84. Shamshad Ali buys a scooter for ₹ 22000. He pays ₹ 4000 cash and agrees to pay the balance in annual installment of ₹ 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

Ans. : Total cost of the scooter = ₹ 22000, Cash paid = ₹ 4000

Balance to be paid = 22000 – 4000 = ₹ 18000

Annual installment = ₹ 1000

$\therefore$ Number of installment = $\frac{18000}{1000} = 18$

Interest of 1st installment = $\frac{18000 \times 10 \times 1}{100}$ = ₹ 1800

Amount of 1st installment = 1000 + 1800 = ₹ 2800

Interest of 2nd installment = $\frac{17000 \times 10 \times 1}{100}$ = ₹ 1700

Amount of 2nd installment = 1000 + 1700 = ₹ 2700

Interest of 3rd installment = $\frac{16000 \times 10 \times 1}{100}$ = ₹ 1600

Amount of 3rd installment = 1000 + 1600 = ₹ 2600

$\therefore$ Sequence of installments is 2800, 2700, 2600, … in A.P

Here, a = 2800, d = 2700 - 2800 = -100 and n = 18

$\therefore S_n = \frac{n}{2}[2a + (n - 1) d] = \frac{18}{2}[2 \times 2800 + (18 - 1) \times (-100)]$

= 9 [5600 – 1700] = ₹ 35100

Therefore, the total cost of tractor is (35100 + 4000) = ₹ 39100

85. Find the sum of the series up to n terms .6 +. 66 + .666+…

Ans. : The given sum is not in GP but we can write it as follows: -

Sum = .6 + .66 + .666 + ...to n terms

= 6(0.1) + 6(0.11) + 6(0.111) + ...to n terms

taking 6 common

= 6[0.1 + 0.11 + 0.111 + ...to n terms]

divide & multiply by 9,we get

= $(\frac{6}{9})$[9(0.1 + 0.11 + 0.111 + ...to n terms)]

= $(\frac{6}{9})$[0.9 + 0.99 + 0.999 + ...to n terms]

$$= \frac{6}{9}\left[\left(\frac{9}{10}\right) + \left(\frac{99}{100}\right) + \left(\frac{999}{1000}\right) + \ldots \text{ to } n \text{ terms}\right]$$

$$= \frac{6}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ldots \text{ to } n \text{ terms}\right]$$

$$= \frac{6}{9}\left[\{1 + 1 + 1 + \ldots \text{ to } n \text{ terms}\} - \left\{\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots \text{ to } n \text{ terms}\right\}\right]$$

$$= \frac{6}{9}\left[n - \left\{\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots \text{ to } n \text{ terms}\right\}\right]$$

Since $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots$ to n terms is in GP with

first term(a) = $\frac{1}{10}$

common ratio(r) = $\frac{10^{-2}}{10^{-1}}$ = 10^{-1} = $\frac{1}{10}$

We know that

Sum of n terms = $\frac{a\left(1 - r^n\right)}{1 - r}$ [As r < 1]

Substituting valuevalue of a & r

$$\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots \text{ to } n \text{ terms } = \frac{a\left(1 - r^n\right)}{1 - r}$$

$$= \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^n\right)}{\left(1 - \frac{1}{10}\right)}$$

$$= \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^n\right)}{\frac{9}{10}}$$

$$= \frac{1\left(1 - 10^{-n}\right)}{9}$$

Therefore, Sum = $\frac{6}{9}\left[n - \frac{1\left(1-10^{-n}\right)}{9}\right]$

86. Find the sum of the series up to n terms 5 + 55 + 555 + ...

Ans. : Now,to find sum = 5 + 55 + 555 + ... n terms.

= $\frac{5}{9}$ [9 + 99 + 999 + ... n terms]

= $\frac{5}{9}$ [(10 - 1) + (100 - 1) + (1000 - 1) + ... n terms]

= $\frac{5}{9}$ [10 + 100 + 1000 ..... – (1 + 1 + ... 1)]

= $\frac{5}{9}$ [10(10^n - 1)/(10 - 1) + (1 + 1 + ... n times)]

= $\frac{50}{81}(10^n - 1) - \frac{5n}{9}$

87. If a and b are the roots $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$ where a, b, c, d form a G.P. Prove that (q + p):(q - p) = 17:15.

Ans. : Let $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ = k

$\therefore \frac{b}{a}$ = k

$\Rightarrow$ b = ak

And $\frac{c}{b}$ = k

$\Rightarrow$ c = bk = (ak)k = ak^2

Also $\frac{d}{c}$ = k

$\Rightarrow$ d = ck = (ak^2)k = ak^3

$\because$ a^a and b^b are the roots $x^2 - 3x + p = 0$

$\therefore$ a + b = $\frac{-(-3)}{1}$ = 3

$\Rightarrow$ a + ak = 3

$\Rightarrow$ a(1 + k) = 3 ...(i)

And ab = $\frac{p}{1}$

$\Rightarrow$ a(ak) = p

$\Rightarrow$ a^2k = p ...(ii)

Also c, d are roots of $x^2 - 12x + q = 0$

$\therefore$ c + d = $\frac{-(-12)}{1}$ = 12

$\Rightarrow$ $ak^2 + ak^3 = 12$

$\Rightarrow$ $ak^2(1 + k)$ = 12 ...(iii)

And c d = $\frac{q}{1}$

$\Rightarrow$ $ak^2(ak^3)$ = q

⇒ $a^2k^5 = q$ ...(iv)

Dividing eq. (iii) by eq. (i), $\frac{ak^2(1+k)}{a(1+k)} = \frac{12}{3}$

⇒ $k^2 = 4$

⇒ $k = \pm 2$

Now $\frac{q+p}{q-p} = \frac{a^2k^5 + a^2k}{a^2k^5 - a^2k} = \frac{a^2k\left(k^4+1\right)}{a^2k\left(k^4-1\right)}$

$= \frac{(\pm 2)^4+1}{(\pm 2)^4-1} = \frac{16+1}{16-1} = \frac{17}{15}$

Therefore, (q + p):(q - p) = 17:15

88. Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2R^n = S^n$.

Ans. : Let the G.P be a, ar, ar^2, ar^3 ....,ar^{n-1}

Here $S = \frac{a\left(r^n-1\right)}{r-1}$

P = a.ar.ar^2 .... $ar^{n-1} = a^n.r^{1+2+3+.......+(n-1)} = a^n.r^{\frac{n(n-1)}{2}}$

and R $= \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + ...\frac{1}{ar^{n-1}} = \frac{r^{n-1}+r^{n-2}+r^{n-3}+..........+1}{ar^{n-1}}$

$= \frac{1(r^n-1)}{r-1}.\frac{1}{ar^{n-1}} = \frac{r^n-1}{ar^{n-1}(r-1)}$

Now $p^2R^n = \frac{a^{2n}.r^{n(n-1)}(r^n-1)^n}{a^n r^{n(n-1)}(r-1)^n} = \frac{a^n(r^n-1)^n}{(r-1)^n} = a^n\left(\frac{r^n-1}{r-1}\right)^n = S^n$

Hence proved.

89. The sum of the first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11, then find the number of terms.

Ans. : Given: a = 11 and $S_4 = 56$

⇒ $S_4 = \frac{4}{2}[2 \times 11 + (4-1)d]$

⇒ 2(22 + 3d) = 56

⇒ 22 + 3d = 28

⇒ 3d = 6

⇒ d = 2

Also, l + (l - d) + (l - 2d) + (l - 3d) = 112

⇒ 4l - 6d = 112

⇒ 4l = 112 + 6d

⇒ $4l = 112 + 6 \times 2$

⇒ 4l = 112 + 12

⇒ 4l = 124

$\Rightarrow$ l = 31

$\therefore$ $a_n = a + (n - 1)d$

$\Rightarrow 31 = 11 + (n-1) \times 2$

$\Rightarrow$ 2(n - 1) = 20

$\Rightarrow$ n - 1 = 10

$\Rightarrow$ n = 11

90. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Ans. : Let the number of terms be 2n then we have the number of odd terms is n

Let the G.P be a, ar, ar^2, ... ar^{2n-1}

Then the odd terms a, ar^2, ar^4, ar^6, ... form a G.P

$\therefore$ $S_{2n} = \frac{a\left(r^{2n}-1\right)}{r-1}$ and $S_n = a\left[\frac{(r^2)^n-1}{r^2-1}\right]$

According to question, $S_{2n} = 5S_n$

$\Rightarrow a\left[\frac{r^{2n}-1}{r-1}\right] = 5a\left[\frac{(r^2)^n-1}{r^2-1}\right]$

$\Rightarrow \frac{1}{r-1} = \frac{5}{r^2-1}$

$\Rightarrow$ r + 1 = 5

$\Rightarrow$ r = 4

91. If A and G be A.M. and G.M. respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

Ans. : Let the two positive numbers be a and b

Therefore $A = \frac{a+b}{2}$ and $G = \sqrt{ab}$

Now, $A \pm \sqrt{(A+G)(A-G)} = A \pm \sqrt{A^2 - G^2}$

$= \frac{a+b}{2} \pm \sqrt{\left(\frac{a+b}{2}\right)^2 - (\sqrt{ab})^2}$

$= \frac{a+b}{2} \pm \sqrt{\frac{a^2+b^2+2ab}{4} - ab}$

$= \frac{a+b}{2} \pm \sqrt{\frac{a^2+b^2+2ab-4ab}{4}}$

$= \frac{a+b}{2} \pm \sqrt{\frac{(a-b)^2}{4}} = \frac{a+b}{2} \pm \frac{a-b}{2}$

$= \frac{a+b}{2} + \frac{a-b}{2}$ and $\frac{a+b}{2} - \frac{a-b}{2}$

$= \frac{a+b+a-b}{2}$ and $\frac{a+b-a+b}{2}$

$= \frac{2a}{2} = a$ and $\frac{2b}{2} = b$

92. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $(3+2\sqrt{2}):(3-2\sqrt{2})$.

Ans. : Let the numbers be a and b

Given: $a + b = 6\sqrt{ab} \Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$

Applying componendo and dividendo, we get

$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{4}{2}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$

Again applying componendo and dividendo, we get

$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

$\Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

Squaring both sides, $\frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$

$\Rightarrow \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

Therefore, the numbers are in the ratio $(3+2\sqrt{2}):(3-2\sqrt{2})$

93. Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

Ans. : $\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$

$\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{a^{\frac{1}{2}}b^{\frac{1}{2}}}{1}$

$a^{n+1} + b^{n+1} = a^{\frac{1}{2}}b^{\frac{1}{2}}\left(a^n + b^n\right)$

$a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b^{n+\frac{1}{2}}$

$a^{n+1} - a^{n+\frac{1}{2}}b^{\frac{1}{2}} = a^{\frac{1}{2}}b^{n+\frac{1}{2}} - b^{n+1}$

$$a^{n+\frac{1}{2}}\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)=b^{n+\frac{1}{2}}\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)$$

$$\left(\frac{a}{b}\right)^{n+\frac{1}{2}}=1$$

$$\left(\frac{a}{b}\right)^{n+\frac{1}{2}}=\left(\frac{a}{b}\right)^{0}$$

$$n+\frac{1}{2}=0$$

$$n=\frac{-1}{2}$$

94. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n + 1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

Ans. : Let a be the first term and r be the common ratio of given G.P.

Then $\frac{\text{Sum of first } n \text{ terms}}{\text{Sum of terms from } (n+1)^{\text{th}} \text{ to } (2n)^{\frac{k}{n^2}}}$

$= \frac{a+ar+ar^2+....+ar^{n-1}}{ar^n+ar^{n+1}+...+ar^{2n-1}}$

$= \frac{a+ar+ar^2+...+ar^{n-1}}{r^n\left[a+ar+ar^2+...+ar^{n-1}\right]} = \frac{1}{r^n}$

95. Find the sum to n terms of the sequences 8, 88, 888, 8888, ...

Ans. : Here Sn = 8 + 88 + 888 + 8888 + ... up to n terms

⇒ Sn = 8(1 +11 + 111 + 1111 + ... up to n terms)

⇒ $S_n = \frac{8}{9}$(9 + 99 + 999 + 9999 + ... up to n terms) ⇒

⇒ $S_n = \frac{8}{9}\left[(10-1)+\left(10^2-1\right)+\left(10^3-1\right)+ \dots \text{ up to } n \text{ terms }\right]$

⇒ $S_n = \frac{8}{9}$[(10 + 10^2 + 10^3 + ... up to n terms) - (1 + 1 + 1 + ... up to n terms)]

⇒ $S_n = \frac{8}{9}\left[\frac{10\times\left(10^n-1\right)}{10-1}-n\right]$

$= \frac{8}{9}\left[\frac{10}{9}\left(10^n-1\right)-n\right]$

$= \frac{80}{81}\left(10^n-1\right)-\frac{8}{9}n$

96. How many terms of G.P. 3, 3^2 , 3^3 ...... are needed to give the sum 120?

Ans. : Here, a = 3 and r = $\frac{3^2}{3} = 3$

$\therefore S_n = \frac{a\left(r^{-}-1\right)}{r-1}$ when r > 1

$\Rightarrow 120 = \frac{3\left(3^{n}-1\right)}{3-1}$

$\Rightarrow 120 = \frac{3}{2}\left(3^{n}-1\right)$

$\Rightarrow 120 \times \frac{2}{3} = 3^{n}-1$

$\Rightarrow 3^n = 81$

$\Rightarrow 3^n = (3)^4$

$\Rightarrow n = 4$

Therefore, the sum of 4 terms of the given G.P. is 120.

97. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Ans. : Let $\frac{a}{r}$, a, r be first three terms of the given G.P.

According to question, $\frac{a}{r} + a + ar = \frac{39}{10}$..........(i)

And $\frac{a}{r} \times a \times r = 1$

$\Rightarrow a^3 = 1$

$\Rightarrow a = 1$

Putting value of a in eq. (i),

$\Rightarrow$ 10+ 10r + 10r^2 = 39r

$\Rightarrow$ 10r^2 - 29r +10 = 0

$\Rightarrow r = \frac{-(-29) \pm \sqrt{(-29)^2 - 4 \times 10 \times 10}}{2 \times 10}$

$\Rightarrow r = \frac{29 \pm \sqrt{841 - 400}}{20}$

$\Rightarrow r = \frac{29 \pm 21}{20}$

Taking $r = \frac{29+21}{20} = \frac{50}{120} = \frac{5}{2}$ and

then the first three terms are $\frac{1}{5/2}, 1, 1 \times \frac{5}{2}$

$\Rightarrow \frac{2}{5}, 1, \frac{5}{2}$

Taking $r = \frac{29-21}{20} = \frac{8}{20} = \frac{2}{5}$

then first three terms are $\frac{1}{2/5}, 1, 1 \times \frac{2}{5}$

$\Rightarrow \frac{5}{2}, 1, \frac{2}{5}$

98. Evaluate: $\sum_{k=1}^{11} (2 + 3^k)$

Ans. : Given: $\sum_{k=1}^{11}\left(2+3^k\right)$

$= (2 + 3^1) + (2 + 3^2) + (2 + 3^3) + (2 + 3^{11})$

$= (2 + 2 + 2 + \ldots\ldots 11 \text{ times}) + (3 + 3^2 + 3^3 + \ldots\ldots + 3^{11})$

$= 22 + (3 + 3^2 + 3^3 + \ldots\ldots + 3^{11})$ ....(i)

Here $3, 3^2, 3^3 \ldots\ldots, 3^{11}$ is in G.P.

$\therefore$ a = 3 and r = $\frac{3^2}{3} = 3$

$$S_n = \frac{3\left(3^{11}-1\right)}{3-1} = \frac{3}{2}\left(3^{11}-1\right)$$

Putting the value of S_n in eq. (i), we get $\sum_{k=1}^{11}\left(2+3^k\right) = 22 + \frac{3}{2}\left(3^{11}-1\right)$

99. Find the sum to indicated number of terms of the geometric progression $x^3, x^5, x^7 \ldots$ n terms (if $x \neq \pm 1$).

Ans. : Here, $a = x^3$ and $r = \frac{x^5}{x^3} = x^2$

$$S_n = \frac{a\left(1-r^n\right)}{1-r} \text{ when } r < 1$$

$$\Rightarrow S_n = \frac{x^3\left[1-\left(x^2\right)^n\right]}{1-x^2}$$

$$\Rightarrow S_n = \frac{x^3}{1-x^2}\left[1-x^{2n}\right]$$

100. The number of terms of an A.P. is even; the sum of odd terms is 24, of the even terms is 30, and the last term exceeds the first by $10\frac{1}{2}$, find the number of terms and the series.

Ans. : Let no. of term be 2n

Odd terms sum $= 24 = T_1 + T_3 + \ldots + T_{2n-1}$

Even terms sum $= 30 = T_2 + T_4 + \ldots + T_{2n}$

Subtract above two equtions

$nd = 6$

$T_{2n} = T_1 + \frac{21}{2}$

$T_{2n} - a = \frac{21}{2}$

$(2n-1)d = \frac{21}{2}$

$12 - \frac{21}{2} = d = \frac{3}{2}$

$\Rightarrow n = 6 \times \frac{2}{3} = 4$

Total terms $= 2n = 8$

Subtite above values in equation of

Sum of even terms or add terms, we get

$a = \frac{3}{2}$

So series is $\frac{3}{2}, 3\frac{9}{2}......$

101. If $a^2,\ b^2,\ c^2$ are in A.P., prove that $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P.

Ans. : $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P if $\frac{b}{a+\ c} - \frac{a}{b+\ c} = \frac{c}{a+\ b} - \frac{b}{a+\ c}$

$\text{LHS} = \frac{b}{a+\ c} - \frac{a}{b+\ c}$

$\Rightarrow \frac{b^2+bc-a^2-ac}{(a+c)(b+c)}$

$\Rightarrow \frac{(b-a)(a+b+c)}{(a+c)(b+c)}\ .....(1)$

$\text{RHS} = \frac{a}{a+b} = \frac{b}{a+c}$

$\Rightarrow \frac{ca+c^2-b^2-ab}{(a+b)(b+c)}$

$\Rightarrow \frac{(c-d)(a+b+c)}{(a+b)(b+c)}\ .....(2)$

and

$a^2,\ b^2,\ c^2$ are in A.P

$\therefore b^2 - a^2 = c^2 - b^2\ .....(3)$

Substituting $b^2 - a^2$ with $c^2 - b^2$

$(1) = (2)$

$\therefore \frac{a}{b+c},\ \frac{b}{a+c},\ \frac{c}{a+b}$ are in A.P

102. There are 25 trees at equal distances of 5 metres in a line with a well, the distance of the well from the nearest tree being 10 metres. A gardener waters all the trees separately starting from the well and he returns to the well after watering each tree to get water for the next. Find the total distance the gardener will cover in order to water all the trees.

Ans. : There are 25 trees at equal distance of 5 m in line with a will (w), and the distance of the well from the nearesst tree = 10 m.

Thus,

The total distance travelled by gardener to tree 1 and back is $2 \times 10\text{m} = 20\text{m}$

Similarly for all the 25 trees.

The distance covred by gardener is

$= 2[10 + (10 + 5) + (10 + 2 \times 5) + (10 + 3 \times 5) + \ ...\ + (10 + 23 \times 5)]$

This froms a seroes of 1st term a = 10, common difference d = 5 and n = 25

$\therefore 10 + (10 + 5) + (10 + 2 \times 5) + \ ...\ + (10 + 23 \times 5)$

$\Rightarrow S_{25} = \frac{25}{2}[2 \times 10 + (24)5] = 25[10 + 60] = 1750\text{m}$

From(1) and (2)
Total distance $= 2 \times 1750\text{m} = 3500\text{m}$

103. A man is employed to count ₹ 10710. he count at the rate od ₹ 180 per minute for half an hour. after this he counts at the rate of ₹ 3 less every minute than the preceding minute. find the time takan by him to count the entire amount.

Ans. : The man of counts at the rate of ₹ 180 per minute for half an hour. After this he counts at the rate of ₹ 3 less every minute than preceding minute.
Then, the amount counted in first 30 mitnute
$= ₹\ 180 \times 30 = ₹\ 5400$
The amount left to be counted aftar 30 minute
$= ₹\ 10710 = 5400 = ₹\ 5310$
ATQ
A.p formed is $(180 - 3) + (180 - 2 \times 3) + \ ... \ = 5310$
Let time takan to count 5310 be t
Then,
$S_t = \frac{t}{2}[200 - 3t]$
$5310 = \frac{t}{2}[200 - 3t]$
or $t = 59$ minute
Thus, the total time takan by the man to count ₹ 10710 is (59 - 30) = 89 minutes.

104. If 10 times the 10th term of an A.P. is equal to 15 times the 15th term, show that 25th term of the A.P. is zero.

Ans. : Given:
$10a_{10} = 15a_{15}$
$10(a + (10 - 1)d) = 15(a + (15 - 1)d)$
$10a + 90d = 15a + 210d$
$5a + 120d = 0$
$a + 24d = 0 \ .....(1)$
$a_{25} = a + (25 - 1)d$
$= a + 24d$
$= 0 \ [\because \text{from}(1) a + 24d = 0]$
Hence proved.

105. How many terms of G.P. $3, \frac{3}{2}, \frac{3}{4} \cdots$ are needed to give the sum $\frac{3069}{512}$?

Ans. : $\text{Sum} = \frac{3069}{512} = \frac{3\left(1 - \frac{1}{2^n}\right)}{\frac{1}{2}}$
$1 - \frac{1}{2^n} = \frac{3069}{512 \times 6} = \frac{1023}{512 \times 2}$
$1 - \frac{1023}{1024} = \frac{1}{2^n}$

$\frac{1}{2^n} = \frac{1}{1024}$

$n = 10$

106. Find the sum: $\sum_{n=1}^{10} \{(\frac{1}{2})^{n-1} + (\frac{1}{5})^{n+1}\}$.

Ans. : $\sum_{n=1}^{10} \left\{ \left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{5}\right)^{n+1} \right\}$

$= \sum_{n=1}^{10} \left(\frac{1}{2}\right)^{n-1} + \sum_{n=1}^{10} \left(\frac{1}{5}\right)^{n+1}$

$= 1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \cdots$

$= \frac{\left(1 - \frac{1}{2^{10}}\right)}{1 - \frac{1}{2}} + \frac{\frac{1}{5}\left(1 - \frac{1}{5^{10}}\right)}{1 - \frac{1}{5}}$

$= \frac{2^{10} - 1}{2^9} + \frac{5^{10} - 1}{5^{11}}$

107. Show that the ratio of the sum of the first n terms of a G.P. to the sum of terms from $(n + 1)^{th}$ to $(2n)^{th}$ terms is $\frac{1}{r^n}$.

Ans. : Sum of first n term of G.P.

$= a + a_2 + a_3 + \ ... \ + a_n$

$= a + ar + ar^2 + \ ... \ + ar^{n-1}$ $[\because t_n = ar^{n-1}]\cdots(i)$

Also sum of term from

$(n + 1)^{th}$ to $(2n)^{th}$ term is

$= a_{n+1} + a_{n+1} + ... \ + a_{2n}$

$= ar^n + ar^{n-1} + \ ... \ + ar^{2n-1}\cdots(i)$

Ratio of (i) and (ii) is:

$= \frac{a + ar + ar^2 + \ ... \ ar^{n-1}}{ar^n + ar^{n-1} + \ ... \ + ar^{2n-1}} \left[\because S_n = \frac{a(1 - r^n)}{1 - r}\right]$

$= \frac{\frac{a(1 - r^n)}{1 - r}}{\frac{ar^n(1 - r^n)}{1 - r}}$

$= \frac{1}{r^n}$

108. The ratio of the sum of first three term is to that of first 6 terms of a G.P. is 125 : 152. Find the common ratio.

Ans. : Let Sum of first three terms = a + ar + ar^2

The ratio $= \frac{a + ar + ar^2}{a + ar + ar^2 + ar^3 + ar^4 + ar^5}$

$= \frac{1 + r + r^2}{1 + r + r^2 + r^3(1 + r + r^2)}\cdots(1)$

Let $A = 1 + r + r^2\cdots(2)$

Ratio $= \frac{A}{A + r^3 A} = \frac{125}{152}$

$\frac{1}{1 + r^3} = \frac{125}{152}$

$152 + 125 + 125r^3$

$r^3 = \frac{27}{125}$

$r = \frac{3}{5}$

109. Find the two numbers whose A.M. is 25 and G.M. is 20.

Ans. : Given,

A.M. = 25

G.M. = 20

Now,

$A.M. = \frac{a+b}{2} = 25$

And, $G.M = \sqrt{ab} = 20$

$a+b = 50, ab = 400$

$(a - b) = \sqrt{(a + b)^2 - 4ab}$

$= \sqrt{(50)^2 - 16000}$

$= \sqrt{2500 - 1600}$

$= \pm 30$

$a-b = \pm 30a+b = 50$ $\quad\quad\quad$ $2a = 80$

$a = 40$

Also, $-2b = -20$

$b = 10$

$\therefore$ The numbers are 40, 10.

110. If S_p denotes the sum of the series $1 + r^p + r^{2r} + \ldots$ to ∞ and S_p the sum of the series $1 - r^p + r^{2p} - \ldots$ to ∞, prove that $S_p + S_p = 2S_{2p}$.

Ans. : $S_p = 1 + r^p + r^{2p} + \ldots + \infty$

$S_p = \frac{1}{1 - r^r}$

$S_p = 1 - r^p + r^{2p} + \ldots + \infty$

$S_p = \frac{1}{1 + r^p}$

Now,

$S_p + S_p = \frac{1}{1 - r^p} + \frac{1}{1 + r^p}$

$= \frac{2}{1 - r^{2p}}$

$S_p + S_p = 2 \times S_{2p}$

111. Find k such that $k+9$, $k-6$ and 4 from three consecutive terms of a G.P.

Ans. : $k+9, k-6, 4$ are in G.P.

$(k-6)^2 = (k+9)4$

$k^2 + 36 - 12k = 4k + 36$

$k^2 - 16k = 0$

$k(k-16) = 0$

$k = 0, k = 16$

112. Prove that: $(9^{\frac{1}{3}}.9^{\frac{1}{9}}.9^{\frac{1}{27}}\ldots\infty) = 3.$

Ans. : $9^{\frac{1}{3}} \times 9^{\frac{1}{9}} \times 9^{\frac{1}{27}}\ldots\infty$

$= 9^{\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ \ldots\right)}$

$= 9^{\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)}$ $\left[\text{Using } S_\infty = \frac{a}{1-r}\right]$

$= 9^{\left(\frac{1}{3}\times\frac{3}{2}\right)}$

$= 9^{\frac{1}{2}}$

$= 3$

So,

$9^{\frac{1}{3}} \times 9^{\frac{1}{9}} \times 9^{\frac{1}{27}}\ldots\infty = 3$

113. Find the sum of the following series to infinit:

$$\frac{1}{3}+\frac{1}{5^2}+\frac{1}{3^3}+\frac{1}{5^4}+\frac{1}{3^5}+\frac{1}{5^6}+\ldots\infty$$

Ans. : The G.P can be written as follows:

$\frac{1}{3}+\frac{1}{5^2}+\frac{1}{3^3}+\frac{1}{5^4}+\frac{1}{3^5}+\frac{1}{5^6}+\ldots\infty$

$= \left(\frac{1}{3}+\frac{1}{3^3}+\frac{1}{3^5}+\cdots\infty\right)+\left(\frac{1}{5^2}+\frac{1}{5^4}+\frac{1}{5^6}+\cdots\infty\right)$

$= \frac{\frac{1}{3}}{1-\frac{1}{3^2}}+\frac{\frac{1}{5^2}}{1-\frac{1}{5^2}}$

$= \frac{3}{8}+\frac{1}{24}$

$= \frac{10}{24}$

$= \frac{5}{12}$

114. If a, b, c, d and p are different real numbers such that:

$(a^2+b^2+c^2)p^2 - 2(ab+bc+cd)p + (b^2+c^2+a^2) \le 0$, then show that a, b, c and d are in G.P.

Ans. : $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + a^2) \le 0,$

$\Rightarrow (a^2p^2 + b^2p^2 + c^2p^2) - 2(abp + bcp + cdp) + (b^2 + c^2 + d^2) \le 0$

$\Rightarrow (a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) \le 0$

$\Rightarrow (ap - b)^2 + (bp - c)^2 + (cp - d)^2 \le 0$

$\Rightarrow (ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$

$\Rightarrow (ap - b)^2 = 0$

$\Rightarrow p = \frac{b}{a}$

Also, $(bp - c)^2 = 0$

$\Rightarrow p = \frac{c}{b}$

Similiarly, $\Rightarrow (cp - d)^2 = 0$

$\Rightarrow p = \frac{d}{c}$

$\therefore \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

Thus, a, b, c and d are in G.P.

115. Find the sum of the following geometric series:

$$\frac{a}{1+i} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \ldots + \frac{a}{(1+i)^n}.$$

Ans. : $\frac{a}{1+i} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \ldots + \frac{a}{(1+i)^n}.$

$a = \frac{a}{1+i}, r = \frac{\frac{a}{(1+i)^2}}{\frac{a}{1+i}} = \frac{1}{1+i}$

$S_n = a\frac{(1-r^n)}{1-r}$

$= \frac{a}{1+i} \times \frac{\left(1 - \left(\frac{1}{1+i}\right)^n\right)}{1 - \frac{1}{1+i}}$

$= \frac{a}{1+i} \times \frac{1+i}{(-i)}(1 - (1+i)^n)$

$= -ai(1 - (1+i)^{-n})$

116. Find the sum of the following series:

0.6 + 0.66 + 0.666 + ... to n terms.

Ans. : $0.6 + 0.66 + 0.666 + \& \ldots$ to n

$= 6 \times 0.1 + 6 \times 0.11 + 6 \times 0.111 + \ldots$

$= \frac{6}{9}\left\{\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots + - \right\}$

$= \frac{6}{9}\left\{\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \ldots + \right\}$

$= \frac{6}{9}\left\{n - \left(\frac{1}{10} + \frac{1}{10^2} + \ldots + \frac{1}{10^n}\right)\right\}$

$$= \frac{6}{9}\left[n - \frac{1}{10}\frac{\{1-(\frac{1}{10})^n\}}{(1-\frac{1}{10})}\right]$$

$$= \frac{6}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

117. If A is the arithmetic mean and G_1 , G_2 be two geometric means between any two numbers, then prove that:

$$2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_2}$$

Ans. : Let the numbers be a and b.

Then, $A = \frac{a+b}{2}$ or $2A = a + b$ .... (1)

Also, G_1 and G_2 are geometric means between a and b, the a, G_1, G_2, b are in G.P.

Let r be the common ratio.

Then, $b = ar^{4-1} = ar^3 \Rightarrow \frac{b}{a} = r^3 \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$

$\therefore\ G_1 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{1}{3}} = a^{\frac{2}{3}}b^{\frac{1}{3}}$

and $G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{3}} = a^{\frac{1}{3}}b^{\frac{2}{3}}$

$\therefore\ \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = \frac{G_1^3 + G_2^3}{G_1G_2} = \frac{a^2b + ab^2}{ab} = a + b = 2A$

118. In a cricket tournament 16 school teams participated. A sum of Rs. 8000 is to be awarded among themselves as prize money. If the last placed team is awarded Rs. 275 in prize money and the award increases by the same amount for successive finishing places, how much amount will the first place team receive?

Ans. : Let the first place team get Rs. a as the prize money.

Since award money increases by the same amount for successive finishing places, we get an A.P.

Let the constant amount be d.

Here, t_{16} = 275, n = 16 and S_{16} = 8000

$\therefore$ t_{16} = a + (16 - 1)(-d)

⇒ 275 = a - 15d ....(1)

Also, $S_{16} = \frac{16}{2}[2a + (n-1)(-d)]$

⇒ 8000 = 8[2a + (16 - 1)(-d)]

⇒ 1000 = 2a - 15d ....(2)

Solving Eqs. (1) and (2), we get a = 725

Hence, first place team recieves Rs. 725.

119. If a_1, a_2, a_3, ..., an are in A.P., where $a_i > 0$ for all i, show that:

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

Ans. : Given that, a1 , a2, ...,an are in A.P., $\forall\ a_i > 0$

$\therefore\ a_1 - a_2 = a_2 - a_3 = ... = a_{n-1} - a_n = -d$ (constant)

Now,

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + ... + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$$

$$= \frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2} + \frac{\sqrt{a_2}-\sqrt{a_3}}{a_2-a_3} + ... + \frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{a_{n-1}-a_n} \text{ (rationalizing)}$$

$$= \frac{\sqrt{a_1}-\sqrt{a_2}}{-d} + \frac{\sqrt{a_2}-\sqrt{a_3}}{-d} + ... + \frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{-d}$$

$$= \frac{1}{-d}\left[\sqrt{a_1} - \sqrt{a_n}\right]$$

$$= \frac{a_1-a_n}{-d\left(\sqrt{a_1}+\sqrt{a_2}\right)} \text{ (rationalizing)}$$

$$= \frac{-(n-1)d}{-d\left(\sqrt{a_1}+\sqrt{a_n}\right)} \text{ (as } a_n = a_1 + (n-1)d)$$

$$= \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

120. Find the r^{th} term of an A.P. sum of whose first n terms is $2n + 3n^2$.
[**Hint:** an = $S_n - S_n - 1$]

Ans. : Given that $S_n = 2n + 3n^2$

$\Rightarrow S_1 = 2n + 3n^2$

$\Rightarrow S_2 = 2 \times 1 + 3(1)^2 = 5$

$\Rightarrow S_3 = 2 \times 3 \times 9 = 33$

$\therefore\ S_1 = a_1 = 5$

$S_2 - S_1 = a_2 = 16 - 5 = 11$

$\therefore\ d = a_2 - a_1 = 11 - 5$

Now $T_r = a_1 + (r - 1)d$

$= 5 + (r - 1)6 = 5 + 6r - 6 = 6r - 1$

Hence, the required rth term is 6r - 1.

121. A man accepts a position with an initial salary of Rs. 5200 per month. It is understood that he will receive an automatic increase of Rs. 320 in the very next month and each month thereafter.

a. Find his salary for the tenth month.
b. What is his total earnings during the first year?

Ans. : The man gets a fixed increment of Rs. 320 each month. Therefore, this forms an A.P. whose,

First term, a = 5200 and Common difference, d = 320

a. Salary for 10^{th} month will be given by an, where n = 10.

∴ Total earning = a_{10}

= a + (n - 1)d

= 5200 + (10 - 1) × 320

= 5200 + 9 × 320

= 5200 + 2880 = Rs. 8080

b. Total earnings during the first year is equal to the sum of 12 terms of the A.P.

∴ Total earnings = S_{12}

$= \frac{12}{2}[2 \times 5200 + (12 - 1)3202]$

= 6[10400 + 11 × 320]

= 6[10400 + 3520]

= 6 × 13920 = Rs. 83520

122. Match the questions given under Column I with their appropriate answers given under the Column II.

	Column I		Column II
(a)	$4, 1, \frac{1}{4}, \frac{1}{16}$	(i)	A.P.
(b)	2, 3, 5, 7	(ii)	Squence
(c)	13, 8, 3, -2, -7	(iii)	G.P.

Ans. :

	Column I		Column II
(a)	$4, 1, \frac{1}{4}, \frac{1}{16}$	(i)	G.P.
(b)	2, 3, 5, 7	(ii)	Squence
(c)	13, 8, 3, -2, -7	(iii)	A.P.

Solution:

1. $4, 1, \frac{1}{4}, \frac{1}{16}$

Here, $\frac{a_2}{a_1} = \frac{1}{4}, \frac{a_3}{a_2} = \frac{\frac{1}{4}}{1} = \frac{1}{4}$ and $\frac{a_4}{a_3} = \frac{a_4}{a_3} = \frac{\frac{1}{16}}{\frac{1}{4}} = \frac{1}{4}$ Hence, it is G.P.

2. 2, 3, 5, 7

Here, $a_2 - a_1 = 3 - 2 = 1$ $a_3 - a_2 = 5 - 3 = 2$ ∴ $a_2 - a_1 \neq a_3 - a_2$ Hence, it is not A.P.

$\frac{a_2}{a_1} = \frac{3}{2}, \frac{a_3}{a_2} == \frac{5}{3}$ So, $\frac{3}{2} \neq \frac{5}{3}$ So, it is not G.P. Hence, it is squence.

3. 13, 8, 3, -2, -7

Here, $a_3 - a_1 = 8 - 13 = -5$ $a_3 - a_2 = 3 - 8 = -5$ So, $a_2 - a_1 = a_3 - a_2 = -5$ So, it is an A.P.

123. If S_1, S_2, S_3 are respectively the sum of $n, 2n$ and $3n$ terms of G.P. then prove that ${S_1}^2 + {S_2}^2 = S_1\left(S_2 + S_3\right)$.

Ans. : Let first term of A.P. be a and common ratio be r.

Then,

$$S_1 = \frac{a\left(r^n - 1\right)}{r-1}, S_2 = \frac{a\left(r^{2n} - 1\right)}{r-1}$$

$$\text{and } S_3 = \frac{a\left(r^{3n} - 1\right)}{r-1}$$

$$S_1 = \frac{a\left(r^n - 1\right)}{r-1}, S_2 = \frac{a\left(r^{2n} - 1\right)}{r-1}$$

$$\text{and } S_3 = \frac{a\left(r^{3n} - 1\right)}{r-1}$$

Now,

$$S_1^2 + S_2^2 = \frac{a^2\left(r^n - 1\right)^2}{(r-1)^2} + \frac{a^2\left(r^{2n} - 1\right)^2}{(r-1)^2}$$

$$= \frac{a^2}{(r-1)^2}\left[\left(r^n - 1\right)^2 + \left(r^{2n} - 1\right)^2\right]$$

$$= \frac{a^2\left(r^n - 1\right)^2}{(r-1)^2}\left\{1 + \left(r^n + 1\right)^2\right\}$$

$$\left[\because r^{2n} - 1 = \left(r^n - 1\right)\left(r^n + 1\right)\right]$$

$$= \frac{a^2\left(r^n - 1\right)^2\left(r^{2n} + 2r^n + 2\right)}{(r-1)^2}$$

$$S_1^2 + S_2^2 = \frac{a^2\left(r^n - 1\right)^2}{(r-1)^2} + \frac{a^2\left(r^{2n} - 1\right)^2}{(r-1)^2}$$

$$= \frac{a^2}{(r-1)^2}\left[\left(r^n - 1\right)^2 + \left(r^{2n} - 1\right)^2\right]$$

$$= \frac{a^2\left(r^n - 1\right)^2}{(r-1)^2}\left\{1 + \left(r^n + 1\right)^2\right\}$$

$$\left[\because r^{2n} - 1 = \left(r^n - 1\right)\left(r^n + 1\right)\right]$$

$$= \frac{a^2\left(r^n - 1\right)^2\left(r^{2n} + 2r^n + 2\right)}{(r-1)^2}$$

and $S_1\left(S_2 + S_3\right) = \frac{a\left(r^n - 1\right)}{r-1}\left[\frac{a\left(r^{2n} - 1\right)}{r-1} + \frac{a\left(r^{3n} - 1\right)}{r-1}\right]$

$$= \frac{a^2}{(r-1)^2}\left(r^n-1\right)\cdot\left[\left(r^{2n}-1\right)+\right.$$

$$\left.\left(r^{3n}-1\right)\right]$$

$$= \frac{a^2\left(r^n-1\right)}{(r-1)^2}\left[\left(r^n-1\right)\left(r^n+1\right)+\right.$$

$$\left[\because a^3-b^3=\left(r^n-1\right)\left(r^{2n}+r^n+1\right)\right]$$

$$= \frac{a^2\left(r^n-1\right)^2}{(r-1)^2}\left.\left(r^{2n}+2b+r^2\right)\right]$$

$$= \frac{a^2}{(r-1)^2}\left(r^n-1\right)\cdot\left[\left(r^{2n}-1\right)+\right.$$

$$\left.\left(r^{3n}-1\right)\right]$$

$$= \frac{a^2\left(r^n-1\right)}{(r-1)^2}\left[\left(r^n-1\right)\left(r^n+1\right)+\right.$$

From equations (i) and (ii)

$$\left[\because a^3-b^3=\left(r^n-1\right)\left(r^{2n}+r^n+1\right)\right]$$

$$= \frac{a^2\left(r^n-1\right)^2}{(r-1)^2}\left.\left(r^{2n}+2b+r^2\right)\right]$$

$$S_1^2+S_2^2=S_1\left(S_2+S_3\right) \quad \text{Hence proved.}$$

$$S_1^2+S_2^2=S_1\left(S_2+S_3\right) \quad \text{Hence proved.}$$

124. For a G.P., if $(m+n)^{\text{th}}$ term is P and $(m-n)$ th term is q, then prove that m^{th} and q^{th} term are $\sqrt{pq}$ and $p\left(\frac{q}{p}\right)^{m/2n}$ respectively.

Ans. : Let first term of sequence be a and common ratio be r, then

$$T_{m+n} = p$$

$$\Rightarrow \quad \text{and} \quad T_{m-n} = q$$

$$\Rightarrow \quad \frac{a\cdot r^{m+n-1}}{} = p$$

$$a\cdot r^{m-n-1} \quad \text{and} \quad a\cdot\frac{p}{q}$$

$T_{m+n} = p$

$\Rightarrow$ and $T_{m-n} = q$

$\Rightarrow \Rightarrow \quad \frac{a \cdot r^{m+n-1}}{r^{2n}} = \frac{p}{q} = p$

$a \cdot r^{m-n-1}$ and $a \cdot \frac{p}{q}$

$$\Rightarrow \quad r = \left(\frac{p}{q}\right)^{1/2n}$$

$$\Rightarrow \quad \frac{1}{r} = \left(\frac{q}{p}\right)^{\frac{1}{2n}}$$

Now, $T_m = a \cdot r^{m-1}$

$$a \cdot r^{m+n-1}\left(\frac{1}{r}\right)^n$$

$$T_{m+n}\left(\frac{1}{r}\right)^n$$

$$= p \cdot \left(\frac{q}{p}\right)^{\frac{n}{2n}}$$

$$\left[\because T_{m+n} = p \text{ and } \frac{1}{r} = \left(\frac{q}{p}\right)^{\frac{1}{2n}}\right]$$

$\Rightarrow \quad T_m = p \cdot \left(\frac{q}{p}\right)^{1/2} = \sqrt{pq}$ Hence proved.

and $T_n = a.r^{n-1}$

$$= a \cdot r^{m+n-1}\left(\frac{1}{r}\right)^m = T_{m+n}\left(\frac{1}{r}\right)^m$$

$= p \cdot \left(\frac{q}{p}\right)^{\frac{n}{2n}}$ $\quad = p \cdot \left(\frac{q}{r}\right)^{\frac{m}{2n}}$

$\left[\because T_{m+n} = p \text{ and } \frac{1}{r} = \left(\frac{q}{p}\right)^{\frac{1}{2n}}\right]$ Hence proved.

$\Rightarrow \quad T_m = p \cdot \left(\frac{q}{p}\right)^{1/2} = \sqrt{pq}$ Hence proved.

and $T_n = a.r^{n-1}$

$= a \cdot r^{m+n-1}\left(\frac{1}{r}\right)^m = T_{m+n}\left(\frac{1}{r}\right)^m$

*** Given section consists of questions of 5 marks each.** **[115]**

125. $= p \cdot \left(\frac{q}{r}\right)^{\frac{m}{2n}}$

Hence proved.

Show that $\dfrac{1 \times 2^2 + 2 \times 3^2 + \ldots \ldots + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + \ldots \ldots + n^2 (n+1)} = \dfrac{3n+5}{3n+1}$

Ans. : Given: $\dfrac{1 \times 2^2 + 2 \times 3^2 + \ldots \ldots + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + \ldots \ldots + n^2 (n+1)} = \dfrac{3n+5}{3n+1}$

$$= \frac{\sum n(n+1)^2}{\sum n^2(n+1)} = \frac{\sum n\left(n^2+2n+1\right)}{\sum\left(n^3+n^2\right)}$$

$$= \frac{\sum n^3 + 2\sum n^2 + \sum n}{\sum n^3 + \sum n^2}$$

$$= \frac{\frac{n^2(n+1)^2}{4} + \frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}}{\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}}$$

$$= \frac{\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{2(2n+1)}{3} + 1\right]}{\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{(2n+1)}{3}\right]}$$

$$= \frac{3n^2+11n+10}{3n^2+7n+2} = \frac{(n+2)(3n+5)}{(n+2)(3n+1)} = \frac{3n+5}{3n+1}$$

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126. Find the sum of the following series up to n terms: $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots\ldots$

Ans. : Given: $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots\ldots$ up to n terms

$$\therefore a_n = \frac{1^3+2^3+3^3+\ldots\ldots+n^3}{1+3+5+\ldots\ldots(2n-1)}$$

$$= \frac{\sum n^3}{\frac{n}{2}[2+(n-1)2]} = \frac{\sum n^3}{\frac{n}{2}(2n)} = \frac{\sum n^3}{n^2} = \frac{n^2(n+1)^2}{4n^2}$$

$$= \frac{1}{4}\left(n^2+2n+1\right)$$

$$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \frac{k^2+2k+1}{4}$$

$= \frac{1}{4}[(1^2 + 2.1 + 1) + (2^2 + 2.2 + 1) + (3^2 + 2.3 + 1) + \ldots\ldots + (n^2 + 2n + 1)]$

$$= \frac{1}{4}\left[\sum n^2 + 2\sum n + n\right]$$

$$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2} + n\right]$$

$$= \frac{n}{4}\left[\frac{2n^2+3n+1+6n+6+6}{6}\right]$$

$$= \frac{n}{24}\left(2n^2+9n+13\right)$$

127. If S_1, S_2, S_3 are the sum of first n natural no. their squares and their cubes respectively, show that $9S_2^2 = S_3(1 + 8S_1)$.

Ans. : $S_1 = \frac{n(n+1)}{2}$

$S_2 = \frac{n(n+1)(2n+1)}{6}$

$S_3 = \left(\frac{n(n+1)}{2}\right)^2$

R.H.S. = $S_3(1 + 8S_1)$

$= \frac{(n(n+1))^2}{2}\left[1 + 8\frac{n(n+1)}{2}\right]$

$= 9\left(\frac{n(n+1)(2n+1)}{6}\right)^2$

$= 9S_2^2$

128. If a, b, c are in A.P.; b, c, d are in G.P. and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P., prove that a, c, e are in G.P.

Ans. : Since, a, b, c are in A.P.

$\therefore$ b - a = c - b

$\Rightarrow$ 2b = a + c

$\Rightarrow b = \frac{a+c}{2}$

Since, b, c, d are in G.P.

$\therefore \frac{c}{b} = \frac{d}{c}$

$\Rightarrow$ c^2 = bd..........(i)

Also $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P.

$\therefore \frac{1}{d} - \frac{1}{c} = \frac{1}{e} - \frac{1}{d}$

$\Rightarrow \frac{2}{d} = \frac{1}{c} + \frac{1}{e}$

$\Rightarrow \frac{2}{d} = \frac{c+e}{ce}$

$\Rightarrow d = \frac{2ce}{c+e}$

Putting values of b and d in eq. (i), $c^2 = \left(\frac{c+a}{2}\right)\left(\frac{2ce}{c+e}\right)$

$\Rightarrow c^2 = \frac{\text{ce }(c+a)}{c+e}$

$\Rightarrow$ $c^2(c + e) = ec(c + a)$

$\Rightarrow$ $c^2 + ce = ce + ae$

$\Rightarrow$ $c^2 = ae$ which shows that a, c, e are in G.P.

129. The ratio of the A.M. and G.M. of two positive numbers a and b is $m:n$. Show that $a:b = (m + \sqrt{m^2 - n^2}):(m - \sqrt{m^2 - n^2})$

Ans. : Given: $\frac{a+b}{2} : \sqrt{ab} = m : n$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$

By componendo and dividendo,

$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n}$

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{m+n}{m-n}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$

Again by componendo and dividendo,

$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$

$\Rightarrow \frac{a}{b} = \frac{(\sqrt{m+n}+\sqrt{m-n})^2}{(\sqrt{m+n}-\sqrt{m-n})^2}$

$\Rightarrow \frac{a}{b} = \frac{m+n+m-n+2\sqrt{(m+n)(m-n)}}{m+n+m-n-2\sqrt{(m+n)(m-n)}}$

$\Rightarrow \frac{a}{b} = \frac{2m+2\sqrt{(m+n)(m-n)}}{2m-2\sqrt{(m+n)(m-n)}}$

$\Rightarrow \frac{a}{b} = \frac{m+\sqrt{(m+n)(m-n)}}{m-\sqrt{(m+n)(m-n)}}$

Therefore, $a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$

130. If a, b, c are in A.P., prove that:

$a^3 + c^3 + 6abc = 8b^3$

Ans. : If $a^3 + c^3 + 6abc = 8c^3$

or $a^3 + c^3 - (2b)^3 + 6abc = 0$

or $a^3 + (-2b)^3 + c^3 + 3 \times a \times (-2b) \times c = 0$

$\therefore (a - 2b + c) = 0 \left[\begin{array}{c} \therefore x^3 + y^3 + z^3 + 3xyz = 0 \\ \text{or if } x+y+z = 0 \end{array} \right]$

or $a+c = 2b$

$a-b = c - b$

and since, a, b, c are in A.P

Thus, $a-b = c - d$

Hence proved. $a^3 + c^3 + 6abc = 8b^3$

131. If a, b, c are in A.P., prove that:

$(a-c)^2 = 4(a-b)(b-c)$

Ans. : If $(a-c)^2 = 4(a-b)(b-c)$

Then,

$a^2 + c^2 - 2ac = 4(ab) - b^2 - ac + bc$

$\Rightarrow a^2 + c^2 4b^2 + 2ac - 4ac - 4bc = 0$

$\Rightarrow (a + c - 2b)^2 = 0$ $\left[\text{ Using } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc\right]$

$\therefore a + c - 2b = 0$

or $a+c = 2b$

and since,

$a,\ b,\ c$ are in A.P [Given]

$a+c = 2b$

Hence proved

$(a-b)^2 = 4(a-b)(b-c)$

132. If a, b, c are in A.P., prove that:

$a^2 + c^2 + 4ac = 2(ab + bc + ca)$

Ans. : If $a^2 + c^2 + 4ac = 2(ab + bc + ca)$

Then,

$a^2 + c^2 + 2ac - 2ab = 2(ab + bc + ca)$

or $(\ a + \ b + \ - c)^2 - \ b^2 = 0$

$[\therefore (\ a + \ b + \ c)^2 = \ a^2 + \ b^2 + \ c^2 + 2\ ab + 2\ ac + 2\ bc]$

or $\ b = \ a + \ c - \ b$

or $2\ b = \ a + \ c$

$b = \frac{a + \ b}{2}$

and since,

a, b, c are in A.P

$b = \frac{a + \ c}{2}$

Thus, $a^2 + \ c^2 + 4\ ac = 2(\ ab + \ bc + \ ca)$

Hence proved.

133. Show that $x^2 + xy + y^2,\ z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P., if x, y and z are in A.P.

Ans. : x, y and z are in A.P.

Let d be the common difference then,

$y = x + d$ and $x = x + 2d$

To show $x^2 + xy + y^2,\ z^2 + zx + x^2 + zx + x^2$ and consecutive terms of an A.P., it is enough to show that,

$(z^2 + zx + x^2) - (x^2 + xy + y^2) = (y^2 + yz + z^2) - (z^2 + zx + x^2)$

LHS $= (z^2 + zx + x^2) - (x^2 + xy + y^2)$

$(z^2 + zx - zy - y^2)$

$= (x2d)^2 + (x + 2d)x - x(x + d) - (x + d)^2$

$= x^2 + 4xd + 4d^2 + x^2 + 2xd - x^2 - xd - x^2 - 2xd - d^2$

$= 3xd + d3^2$

RHS $= (y^2 + yz + z^2) - (z^2 + zx + x^2)$

$= (y^2 + yz + z^2) - (z^2 + zx + x^2)$

$= (x + d)^2 + (x + d)(x + 2d) - (x + 2d)x = x^2$

$= x^2 + 2dx + d^2 + x^2 + 2dx + xd + 2d^2 - x^2 - 2dx - x^2$

$= 3xd + 3d^2$

$\therefore$ LHS = RHS

$\therefore x^2 + xy + y^2,\ z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P.

134. Show that $x^2 + xy + y^2,\ z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P., if x, y and z are in A.P.

Ans. : x, y and z are in A.P.

Let d be the common difference then,

$y = x + d$ and $x = x + 2d$

To show $x^2 + xy + y^2,\ z^2 + zx + x^2 + zx + x^2$ and consecutive terms of an A.P., it is enough to show that,

$(z^2 + zx + x^2) - (x^2 + xy + y^2) = (y^2 + yz + z^2) - (z^2 + zx + x^2)$

LHS $= (z^2 + zx + x^2) - (x^2 + xy + y^2)$

$(z^2 + zx - zy - y^2)$

$= (x2d)^2 + (x + 2d)x - x(x + d) - (x + d)^2$

$= x^2 + 4xd + 4d^2 + x^2 + 2xd - x^2 - xd - x^2 - 2xd - d^2$

$= 3xd + d3^2$

RHS $= (y^2 + yz + z^2) - (z^2 + zx + x^2)$

$= (y^2 + yz + z^2) - (z^2 + zx + x^2)$

$= (x + d)^2 + (x + d)(x + 2d) - (x + 2d)x = x^2$

$= x^2 + 2dx + d^2 + x^2 + 2dx + xd + 2d^2 - x^2 - 2dx - x^2$

$= 3xd + 3d^2$

$\therefore$ LHS = RHS

$\therefore x^2 + xy + y^2,\ z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P.

135. If a, b, c are in A.P., prove that:

$(a - c)^2 = 4(a - b)(b - c)$

Ans. : If $(a - c)^2 = 4(a - b)(b - c)$

Then,

$a^2 + c^2 - 2ac = 4(ab) - b^2 - ac + bc$

$\Rightarrow a^2 + c^2 4b^2 + 2ac - 4ac - 4bc = 0$

$\Rightarrow (a + c - 2b)^2 = 0$ [Using $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$]

$\therefore a + c - 2b = 0$

or $a+c = 2b$

and since,

a, b, c are in A.P [Given]

$a+c = 2b$

Hence proved

$(a - b)^2 = 4(a - b)(b - c)$

136. Find the sum of all two digit numbers which when divided by 4, yields 1 as remainder.

Ans. : sum of all two digit numbers which when divided by 4,
yields 1 as remainder,

$\Rightarrow$ all $4n + 1$ terms with $n \geq 3$

$n = 22, a = 13, d = 4$

Sumof terms $= \frac{22}{2}[26 + 21 \times 4] = 11 \times 110 = 1210$

137. A man accepts a position with an initial salary of ₹ 5200 per month. It is understood that he will receive an automatic increase of ₹ 320 in the very next month and each month thereafter.

i. Find his salary for the tenth month.

ii. What is his total earnings during the first year?

Ans. : A man accepts a position with an initial salary of ₹ 5200 per month.

$a = 5200$

Man w i 11 receive an au tom ati c increase of ₹ 320.

$d = 320$

Man's sa I ary for the n" month is given by,

$a_n = a_1 + (n - 1)d$

Total earnig of the man for the first year

$= \frac{12}{2}[a_1 + a_{12}]$

$= 6[5200 + 5200 + (12 - 1)320]$

$= 83520$

Total earnlg of the man for the first year Is ₹ 83,520.

138. If a, b, c, d are in G.P., prove that:

$(b + c)(b + d) = (c + a)(c + d)$

Ans. : a, b and c are in G.P.

$\therefore b^2 = ac \cdots(1)$

$L.H.S = (b+a)(b + d)$

$= b^2 + bd + bc + cd$

$= ac + c^2 + ad + cd$ [Using (1)]

$= c(a + c) + d(a + c)$

$= (c + a)(c + d)$

$=$ R.H.S

$\therefore$ R.H.S = L.H.S

139. If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots $x^2 - 12x + q = 0$, where a, b, c, d from a G.P. Prove that (q + p) : (q - p) = 17 : 15.

Ans. : Given,

a, b are roots of the equation $x^2 - 3x + p = 0$

$\Rightarrow a + b = 3, ab = p$

And c, d are roots of the equation $x^2 - 12x + q = 0$

$\Rightarrow c + d = 12, cd = q$

Let b = ar, c = ar2 and d = ar3, then a + b = 3 and c + d = 12

$a(1 + r) = 3$ and $ar^2(1 + r) = 12$

$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{12}{3}$

$\Rightarrow r = 2$

And $a(r + 2) = 3$

$\Rightarrow a = 1$

$p = ab$

$p = a \times ar$

$p = 2$

$q = cd$

$= ar^2 \times ar^3$

$a = 32$

$\frac{q+p}{q-p} = \frac{32+2}{32-2}$

$= \frac{34}{30}$

$(q + p) : (q - p) = 17 : 15$

140. The product of three numbers in G.P. is 125 and the sum of their products taken in pairs is $87\frac{1}{2}$. Find them.

Ans. : Let the three number in G.P. be $\frac{a}{r}$, a, ar then product of these numbers $\left(\frac{a}{r}\right)(a)(ar)$

$\Rightarrow a^3 = 125 = 5^3$

$a = 5$

Also, sum of these product in pair

$\left(\frac{a}{r}\right)(a) + (a)(ar) + \left(\frac{a}{r}\right)(ar)$

$= 87\frac{1}{2} = \frac{195}{2}$

$= (5)^2\left(\frac{1+r^2+r}{r}\right) = \frac{195}{2}$

$1 + r^2 + r = \left(\frac{195}{2\times 25}\right)^r$

$2(1 + r^2 + r) = \frac{39}{5}r$

$10 + 10r^2 + 10r = 39r$

$10r^2 - 25r - 4r + 10 = 0$

$5r(2r - 5) - 2(2r - 5) = 0$

$r = \frac{5}{2}, \frac{2}{5}$

$\therefore$ G.P. is $\frac{a}{r}$, a, ar

$10, 5, \frac{5}{2}, \ ... \text{or} \frac{5}{2}, 5, 10 \ ...$

141. If a, b, c are in G.P., prove that:

$(a + 2b + 2c)(a - 2b + 2c) = a^2 + 4c^2$

Ans. : a, b, c are in G.P.

a, b = ar, c = ar^2

$\text{L.H.S} = (a+2b + 2c)(a - 2ar + 2c)$

$= \left(a + 2ar + 2ar^2\right)\left(1 - 2ar + 2ar^2\right)$

$= a^2\left(1 + 2ar + 2ar^2\right)\left(1 - 2r + 2r^2\right)$

$= a^2\left[\left(1 + 2r^2\right)^2 - (2r)^2\right]$

$= a^2\left[1 + 4r^4 + 4r^2 - 4r^2\right]$

$= a^2\left[1 + 4r^4\right]$

$= a^2 + 4\left(ar^2\right)^2$

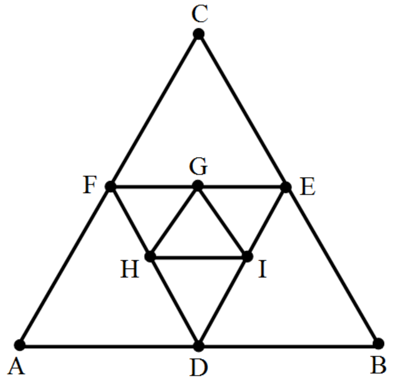
$= a^2 + 4c^2$

$= \text{R.H.S}$

$\therefore \text{R.H.S} = \text{L.H.S}$

142. One side of equilateral triangle is 18 cm. The mid-points of its sides are joined to from another triangle whose mind-points, in turn, are joined to from still another triangle. the process is continued indefinitely. Find the sum of the (i) Perimeters of all the triangles. (ii) Areas of all triangles.

Ans. :



Side of triangle = 18cm.

AD = BD = 9cm.

DE = BD = 9cm.

$GI = IF = \frac{9}{2}cm.$

Sides of the triangles are $18, 9, \frac{9}{2} \ldots$

(i) Sum of perimeters of the equilateral triangle $= \left(54 + 27 + \frac{27}{2} + \ \ldots\right)$

$= \frac{54}{1 - \frac{1}{2}}$

$= 54 \times 2$

Perimeter = 108cm.

(ii) Sum of area of equilateral triangle

$= \left[\frac{\sqrt{3}}{4}(18)^2 + \frac{\sqrt{3}}{4}(9)^2 + \frac{\sqrt{3}}{4}\left(\frac{9}{2}\right)^2 + \ldots\right]$

$= \frac{\sqrt{3}}{4}\left[324 + 81 + \frac{81}{4} + \ldots\right]$

$= \frac{\sqrt{3}}{4}\left[\frac{324}{1 - \frac{1}{4}}\right]$

$= \frac{\sqrt{3}}{4}\left[\frac{324 \times 4}{3}\right]$

$= \sqrt{3}(108)$

143. If a, b, c, are in G.P., prove that:

$(a^2 + b^2), (b^2 + c^2), (c^2 + d^2)$ are in G.P.

Ans. : a, b, c, d are in G.P.

$\therefore b^2 = ac$

$ab = bc$

$c^2 = bd \cdots (1)$

Now,

$(b^2 + c^2)^2 = (b^2)^2 2b^2c^2 + (c^2)^2$

$\Rightarrow (b^2 + c^2)^2 = (ac)^2 + b^2c^2 + b^2c^2 + (bd)^2$ [Using (1)]

$\Rightarrow (b^2+c^2)^2 = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$ [Using (1)]

$\Rightarrow (b^2+c^2)^2 = a^2(c^2+d^2) + b^2(c^2+d)^2$

$\Rightarrow (b^2+c^2)^2 = (a^2+b^2)(c^2+d^2)$

$\therefore (a^2+b^2), (c^2+d^2)$ and (b^2+c^2) are also in G.P.

144. If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y and z respectiveiy. Prove that x, y, z are in G.P.

Ans. : $a_4 = x$

$\Rightarrow ar^3 = x$

Also, $a_{16} = y$

$\Rightarrow ar^9 = y$

And, $a_{16} = z$

$\Rightarrow ar^{15} = z$

$\because \frac{z}{x} = \frac{ar^9}{ar^3} = r^6$

And, $\frac{z}{y} = \frac{ar^{15}}{ar^9} = r^6$

$\therefore \frac{y}{x} = \frac{z}{y}$

$\therefore$ x, y and z are in G.P.

145. Find the 4^{th} term from the end of the G.P. $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}\frac{1}{54}, \ldots, \frac{1}{4374}$.

Ans. : $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}\frac{1}{54}, \ldots, \frac{1}{4374}$.

$a = \frac{1}{2}, l = \frac{1}{4374}, r = \frac{t_{n-1}}{t_n} = \frac{t_2}{t_1} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$

Term from the end is:

$a_n = l\left(\frac{1}{r}\right)^{n-1}$

$t_4 = \left(\frac{1}{4374}\right)(3)^{n-1}$

$= \frac{1}{4374} \times 3^3$

$= \frac{1}{162}$

$\therefore$ 4^{th} term from the end is $= \frac{1}{162}$

146. If the p^{th} and qth terms of a G.P. are q and p respectively, show that its $(p + q)^{th}$ term is $\left(\frac{q^p}{p^q}\right)\frac{1}{p-q}$.

Ans. : Let the first term and common ratio of G.P. are a and r, respectively.

Given that, p^{th} term = q $\Rightarrow ar^{p-1} = q$ ....(1)

and q^{th} term = p ⇒ $ar^{q-1} = p$ ....(2)

On dividing Eq. (1) by (2), we get,

$$\frac{ar^{p-1}}{ar^{q-1}} = \frac{q}{p}$$

$$\Rightarrow r^{p-q} = \frac{q}{p}$$

$$\Rightarrow r = \left(\frac{q}{p}\right)^{\frac{1}{p-q}}$$

On substituting the value of r in Eq. (1), we get

$$a\left(\frac{q}{p}\right)^{\frac{p-1}{p-q}} = q$$

$$\Rightarrow a = q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}}$$

∴ (p + q)th term, $T_{p+q} = a \cdot r^{p+q-1}$

$$= q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}}\left(\frac{a}{p}\right)^{\frac{p+q-1}{p-q}}$$

$$= q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q} - \frac{p+q-1}{p-q}}$$

$$= q\left(\frac{q}{p}\right)^{\frac{q}{p-q}}$$

$$= \frac{q^{\frac{q}{p-q}+1}}{p^{\frac{q}{p-q}}}$$

$$= \frac{q^{\frac{p}{p-q}}}{p^{\frac{q}{p-q}}}$$

$$= \left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$$

147. Match the questions given under Column I with their appropriate answers given under the Column II.

	Column I		Column II
(a)	$1^2 + 2^2 + 3^2 + \ldots + n^2$	(i)	$[\frac{n(n+1)}{2}]^2$
(b)	$1^3 + 2^3 + 3^3 + \ldots n^3$	(ii)	$n(n+1)$
(c)	$2 + 4 + 6 + \ldots + 2n$	(iii)	$\frac{n(n+1)(2n+1)}{6}$
(d)	$1 + 2 + 3 + \ldots n$	(iv)	$\frac{n(n+1)}{2}$

Ans. :

	Column I		Column II
(a)	$1^2 + 2^2 + 3^2 + \ldots + n^2$	(iii)	$\frac{n(n+1)(2n+1)}{6}$
(b)	$1^3 + 2^3 + 3^3 + \ldots n^3$	(i)	$\left[\frac{n(n+1)}{2}\right]^2$
(c)	$2 + 4 + 6 + \ldots + 2n$	(ii)	$n(n+1)$

(d)	$1 + 2 + 3 + \ldots . n$	(iv)	$\frac{n(n+1)}{2}$

Solution:

1. Let $S = 1^2 + 2^2 + 3^2 + \ldots . + n^2$

We have, $n^3 - (n-1)^3 = 3n^2 - 3n + 1$; And by changing n into n - 1, $(n-1)^3 - (n-2)^3 = 3(n-1)^2 - 3(n-1) + 1$; $(n-2)^2 - (n-3)^3 = 3(n-2)^2 - 3(n-2) + 1$; .... $3^3 - 2^3 = 3.3^2 - 3.3 + 1$; $2^3 - 1^2 = 3.2^2 - 3.2 + 1$; $1^2 - 0^2 = 3.1^2 - 3.1 + 1$ Hence, by addition, $n^3 = 3(1^2 + 2^2 + 3^2 + \ldots . + n^2) - 3(1 + 2 + 3 + \ldots . + n) + n$

$= 3s - \frac{3n(n+1)}{2} + n$ $\Rightarrow 3S = n^3 - n + \frac{3n(n+1)}{2} = n(n+1)\left(n - 1 + \frac{3}{2}\right)$

$\Rightarrow S = \frac{n(n+1)(2n+1)}{6}$

2. Let $S = 1^3 + 2^3 + 3^3 + \ldots .. + n^3$

We have, $n^4 - (n-1)^4 = 4n^3 - 6n^2 + 4n - 1$; $(n-1)^4 - (n-2)^4 = 4(n-1)^3 - 6(n-1)^2 + 4(n-1) - 1$; $(n-2)^4 - (n-3)^4 = 4(n-2)^3 - 6(n-2)^2 + 4(n-2) - 1$; .... $3^4 - 2^4 = 4.3^3 - 6.3^2 + 4.3 - 1$; $2^4 - 1^4 = 4.2^3 - 6.2^2 + 4.2 - 1$; $1^4 - 0^4 = 4.1^3 - 6.1^2 + 4.1 - 1$. Hence, by addition, $n^4 = 4S - 6\left(1^2 + 2^2 + \ldots . + n^2\right) + 4(1 + 2 + \ldots . + n) - n$;

$\therefore\ 4S = n^4 + n + 6\left(1^2 + 2^2 + \ldots . + n^2\right) - 4(1 + 2 + \ldots . + n)$

$= n^4 + n + n(n+1)(2n+1) - 2n(n+1)$ $= n(n+1)\left(n^2 - n + 1 + 2n + 1 - 2\right)$

$= n(n+1)\left(n^2 + n\right)$ $\therefore\ S = \frac{n^2(n+1)^2}{4} = \left\{\frac{n(n+1)}{2}\right\}^2$

3. $1 + 2 + 3 + \ldots . + n = \frac{n(n+2)}{2}$

$2 + 4 + 6 + \ldots . + 2n = 2(1 + 2 + 3 + \ldots . + n) = 2 \times \frac{n}{2}(1 + n) = n(n+1)$

4. 1 + 2 + 3 + ... + n = Sum of n terms of A.P. with first term '1' and common difference ´1´ $= \frac{n}{2}(1 + n)$

*** Case study based questions** **[8]**

148. A company produces 500 computers in the third year and 600 computers in the seventh year. Assuming that the production increases uniformly by a constant number every year.

Based on the above information, answer the following questions.

(i) The value of the fixed number by which production is increasing every year is

(a) 25 (b) 20 (c) 10 (d) 30

(ii) The production in first year is

(a) 400 (b) 250 (c) 450 (d) 300

(iii) The total production in 10 years is

(a) 5625 (b) 5265 (c) 2655 (d) 6525

(iv) The number of computers produced in 21 st year is

(a) 650 (b) 700 (c) 850 (d) 950

(v) The difference in number of computers produced in 10th year and 8th year is

(a) 25 (b) 50 (c) 100 (d) 75

Ans. : (i) Since, it is given that, production increases uniformly by a constant number, hence number of productions every year forms an AP.

$\therefore a_3 = 500 \Rightarrow a + 2d = 500$

$a_7 = 600 \Rightarrow a + 6d = 600$

Now, subtracting Eq. (i) from Eq. (ii), we get

$4d = 100 \Rightarrow d = 25$

(ii) Put $d = 25$ in Eq. (i), we get

$a + 50 = 500 \Rightarrow a = 450$

(iii) The total production in 10 years $= S_{10}$

$$\therefore S_{10} = \frac{10}{2}[2 \times 450 + 9 \times 25]$$
$$= 5[900 + 225] = 5625$$

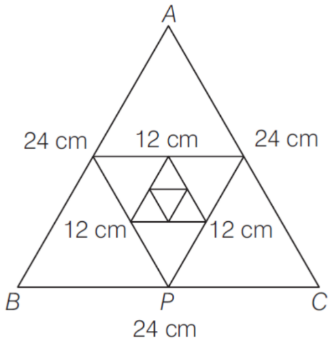
(iv) The number of computers produced in 21st

year $= a_{21}$

$\therefore a_{21} = 450 + 20 \times 25 = 450 + 500 = 950$

(v) $a_{10} - a_8 = (a + 9d) - (a + 7d) = 2d = 2 \times 25 = 50$

149. Each side of an equilateral triangle is 24 cm. The mid-point of its sides are joined to form another triangle. This process is going continuously infinite.



Based on above information, answer the following questions.

(i) The side of the 5th triangle is (in cm)

(a) 3 (b) 6 (c) 1.5 (d) 0.75

(ii) The sum of perimeter of first 6 triangle is (in cm)

(a) $\frac{569}{4}$ (b) $\frac{567}{4}$ (c) 120 (d) 144

(iii) The area of all the triangle is (in sq cm)

(a) 576 (b) $192\sqrt{3}$ (c) $144\sqrt{3}$ (d) $169\sqrt{3}$

(iv) The sum of perimeter of all triangle is (in cm)

(a) 144 (b) 169 (c) 400 (d) 625

(v) The perimeter of 7 th triangle is (in cm)

(a) $\frac{7}{8}$ (b) $\frac{9}{8}$ (c) $\frac{5}{8}$ (d) $\frac{3}{4}$

Ans. : (i) Side of first triangle is 24 .

Side of second triangle is $\frac{24}{2} = 12$

Similarly, side of second triangle is $\frac{12}{2} = 6$

$\therefore a = 24, r = \frac{12}{24} = \frac{1}{2}$

$\therefore$ Side of the fifth triangle,

$$a_5 = ar^4 = 24 \times \left(\frac{1}{2}\right)^4$$

$$= \frac{24}{16} = \frac{3}{2} = 1.5 \text{ cm}$$

(ii) $= \dfrac{72\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\frac{1}{2}} = \dfrac{72 \times 63 \times 2}{2^6}$

$= \dfrac{567}{4}$ cm

(iii) Area of first triangle is $\frac{\sqrt{3}}{4}(24)^2$

Area of second triangle $= \frac{\sqrt{3}}{4}\left(\frac{24}{2}\right)^2 = \frac{\sqrt{3}}{4}(24)^2 \times \frac{1}{4}$

$\therefore a = \frac{\sqrt{3}}{4}(24)^2, r = \frac{1}{4}$

$\therefore$ Sum of area of all triangles

$= \dfrac{a}{1-r} = \dfrac{\sqrt{3}}{4}\dfrac{(24)^2}{1-\frac{1}{4}}$

$= \dfrac{\sqrt{3} \times (24)^2}{3} = 192\sqrt{3}$ cm^2

(iv) The sum of perimeter of all triangle $3(24+12+6+\cdots)$ is

$3\left(\dfrac{24}{1-\frac{1}{2}}\right) = 144$ cm $\left[\because a = 24, r = \frac{1}{2}\right]$

(v) Here, $a = 72, r = \frac{1}{2}$

$a_7 = (72)\left(\frac{1}{2}\right)^6$

$= \dfrac{72}{64} = \dfrac{9}{8}$ cm

----- “Opportunities don’t happen, you create them. -----